Economic Issues, Vol. 14, Part 1, 2009

The Theoretical Analysis of Income Tax Evasion Revisited

Emin Gahramanov[1]

ABSTRACT

There exists an important puzzle in the income tax evasion literature, that higher tax rates unambiguously encourage tax compliance (Yitzhaki's (1974) result). In this paper I show that by simply relaxing the assumption that higher tax rates necessarily translate into higher penalty payments at the initial optimum when caught, it is possible to generate rigorously an important extra disincentive to compliance, which is absent from the previous works. It can also be shown that Yitzhaki's (1974) result is a special case of the present formulation.

1. INTRODUCTION

THE FORMAL ECONOMIC ANALYSIS of income tax evasion was pioneered by Allingham and Sandmo (1972), where a rational and amoral taxpayer maximizes expected utility, which solely depends on income. When caught, the agent must pay penalties, imposed on the amount of evaded *income*. A key comparative static result is that when the tax rate goes up, competing income and substitution effects might lead to more or less tax compliance. The substitution effect encourages evasion since the marginal benefit of cheating goes up with the tax rate. On the contrary, the income effect tends to suppress evasion since a higher tax rate makes the taxpayer with decreasing absolute risk aversion feel worse-off, and thus decrease risk-taking. Therefore, the net effect is ambiguous.

However, Shlomo Yitzhaki (1974) showed that when the penalty is imposed on the amount of evaded *taxes*, as it is under most current tax laws, the substitution effect vanishes. At the original optimum, the penalty paid on concealed income increases proportionally with the tax rate, and hence, there is no substitution effect. The remaining income effect is responsible for inducing the taxpayer to cheat less.[2] Therefore, the net effect is better compliance. Yitzhaki's (1974) result is perhaps the single most important finding in the early tax evasion literature, having spurred a lot of remarkable extensions and discussions. It often becomes a subject of harsh criticism[3] and induces some authors to abandon the expected utility approach to the analysis of the income tax evasion phenomenon.[4]

In this paper I show that it is premature to disregard the basic expected utility-based formulation of the income tax evasion problem in order to obtain a sensible theoretical relationship between taxes and income declaration incentives. Of course, there is already no lack of explanations to the puzzling positive relationship between the tax rate and honesty incentives,[5] and I am not arguing against them. It is worth mentioning, however, that those explanations often depart from Yitzhaki's (1974) initial framework fairly significantly. Instead, my strategy is to adopt as few new assumptions as possible. Namely, I revisit the assumption that an increase in the tax rate necessarily translates into an increase in total penalty payments at the initial level of declared income, which is bound to be the case in Yitzhaki's (1974) formulation, where the relative tax rate increase is uniform for any income level. To the best of my knowledge, this theoretical assumption is implicit in all previous studies on income tax evasion, revisiting Yitzhaki's (1974) puzzle. But there is no a priori reason to assume that in a complex real-world, tax rates would increase in such a way, meaning that a higher tax rate policy can well lead to lower penalty payments at the previous optimum, which would encourage evasion. Finally, it will follow that Yitzhaki's (1974) result is a special case of my formulation. I now turn to the model.

2. The model

In the spirit of Allingham-Sandmo (1972) and Yitzhaki (1974), let w and x stand for true and declared income, respectively. The risk-averse taxpayer's problem is to maximise his expected utility:

$$E[U] = (1-p)U(w-t(x)) + pU(w-t(x)-F(t(w)-t(x))) \tag{1}$$

by optimally choosing x. Here p is the likelihood of getting caught, F is the fine rate ($F > 1$), and $t(x)$ is a general well-behaved tax function with $t'(x) > 0$ and $t''(x) > 0$.[6] The symbols ' and '' stand for the corresponding first and second derivatives, respectively. The agent with zero income owes no taxes to the government, like in Yitzhaki's (1974) specification. It is obvious that the term $F(t(w) - t(x))$ stands for the total penalty payment when caught.

Thus, the first-order condition is:

$$\frac{\partial E[U]}{\partial x} = -(1-p)U'(Y)t'(x) - pU'(Z)(1-F)t'(x) = 0 \tag{2}$$

where $Y \equiv w - t(x)$ and $Z \equiv w - t(x) - F(t(w) - t(x))$. Expression (2) can be re-written as $-(1-p)U'(Y) - p(1-F)U'(Z) = 0$, which is consonant with Yitzhaki (1974). Problem (1) leads to the unique solution for the choice variable. Namely, observe that:

$$\frac{\partial^2 E[U]}{\partial x^2} \equiv D = (1-p)\left[U^{''}(Y)(t'(x))^2 - U^{'}(Y)t^{''}(x)\right] + p\left[U^{''}(Z)((1-F)t'(x))^2 + U^{'}(Z)(F-1)t^{''}(x)\right] \tag{3}$$

Utilising the first-order condition reduces (3) to:

$$D = (1-p)U^{''}(Y)(t'(x))^2 + pU^{''}(Z)((1-F)t'(x))^2 \tag{3'}$$

which is always negative.

It is straightforward to show that the conditions for an interior solution can be simplified as:

$$\frac{U^{'}(w)}{U^{'}(w-Ft(w))} < \frac{p(F-1)}{1-p} \tag{4}$$

and

$$pF < 1 \tag{5}$$

Equations (4) and (5) are identical to conditions (5')* and (6')*, respectively, in Yitzhaki (1974) with $\theta w = t(w)$.

Further, from (2) and (3') it can be established that:

$$\frac{\partial x}{\partial F} = p\frac{t^{'}(x)}{D}\left[U^{''}(Z)(t(w)-t(x))(F-1) - U^{'}(Z)\right] > 0 \tag{6}$$

and

$$\frac{\partial x}{\partial p} = -\frac{t^{'}(x)}{D}\left[U^{'}(Y) + U^{'}(Z)(F-1)\right] > 0 \tag{7}$$

These are familiar results, specifically that heavier penalties and more aggressive monitoring encourage honesty.

To find out the relationship between the tax rate increase and declared income, and to consider a non-uniform variation in the tax schedule, present the tax function for the initial amount of declared income and the true income level as $t(x) + f(x)$ and $t(w) + v(w)$, where $f(x)$ and $v(w)$ are the respective income-dependent shift functions. Now, for the sake of illustration take $f(x)$ and $v(x)$ as τx and $\varepsilon\tau w$, respectively, where τ is a shift parameter, augmented in the latter case by a constant $\varepsilon \leq 1$. Consider now

1. $\varepsilon = 1$ (Yitzhaki's (1974) case). Differentiating (2) with respect to τ (and then evaluating the result at $\tau = 0$), recalling that $-(1-p)U'(Y) - p(1-F)U'(Z)$ is zero, and performing some algebra, results in:

$$\left.\frac{\partial x}{\partial \tau}\right|_{\tau=0} = -\frac{t'(x)}{D}U'(Y)(1-p)\left\{x\left[R_A(Z)-R_A(Y)\right]+F(w-x)R_A(Z)\right\} \tag{8}$$

where $R_A(\bullet) \equiv -U''(\bullet)/U'(\bullet)$. Note that $R_A(Y) < R_A(Z)$. It follows that (8) is positive. In Yitzhaki's (1974) formulation, $t'(x) = \theta$, which would also make (8) positive.[7]

2. $\varepsilon < 1$, and is small. Consequently, equation (8) becomes

$$\left.\frac{\partial x}{\partial \tau}\right|_{\tau=0} = -\frac{t'(x)}{D}U'(Y)(1-p)\left\{x\left[R_A(Z)-R_A(Y)\right]+F(\varepsilon w-x)R_A(Z)\right\} \tag{8'}$$

The sign of (8') is clearly ambiguous.[8] Intuitively, when an increase in the tax rate at the taxpayer's optimum is associated with a smaller relative rise in the endpoint of the tax rate bracket, then total penalty payments actually decrease at the initial amount of declared income, creating more incentives to tax evasion than before[9] (it follows that the impact on the taxes evaded is less clear-cut).[10] This extra disincentive to income declaration is absent from the previous analyses where the tax rate shift is uniform for any income level, naturally causing total penalty payments at the initial optimum to necessarily go up with the tax rate. There is no *a priori* reason whatsoever to believe that this is always the case in reality. Previous studies overlook this complication, which, nevertheless, helps to restore the original ambiguity in the Allingham-Sandmo (1972) expected-utility framework in a very simple way.

3. Conclusion

Yitzhaki's (1974) formulation of the dishonest taxpayer's problem generated a puzzle that a higher tax rate encourages income declaration, which is contrary to most empirical evidences and economic intuition. Although various remarkable contributions (predominantly deviating from the original analysis quite significantly), reconciled the puzzle within the neoclassical paradigm, my approach in this paper was to accomplish the same goal with a minimum set of extra assumptions. I showed that this can be done by revisiting the assumption that total penalty payments at the original optimum necessarily go up with a change in the tax policy. Since I allowed only a slight departure from the original framework (without appealing to labour-leisure choice, honesty characteristics, prospect theory, etc.), I conclude that the tax evasion framework of Allingham-Sandmo and Yitzhaki (1974) is legitimate, and it is premature to disregard it based on earlier finding that higher taxes encourage compliance. Nevertheless, it will be interesting to know what additional light future empirical studies can shed on the validity of the analytical presumptions I make in this paper.

Date of acceptance: 21 July 2008

ENDNOTES

1. School of Accounting, Economics and Finance at Deakin University. Corresponding address: Deakin University, Faculty of Business and Law, 221 Burwood Highway, Burwood, Victoria 3125, Australia. Phone: + 61 3 925 17467.
E-mail: eming@deakin.edu.au.
This paper is a modified excerpt from a thesis submitted to the Academic Faculty of Colorado State University in partial fulfillment of the requirements for the degree of the Doctor of Philosophy in Economics. The author thanks two anonymous referees and the editors for helpful suggestions. Any error is the responsibility of the author.

2. Interestingly, although most empirical studies reveal that higher tax rates encourage evasion (see, among others, Clotfelter (1983), Crane and Nourzad (1990), Alm, Bahl and Murray (1993) and references therein), a few studies find just the opposite. For more details refer to Geeroms and Wilmots (1985) and Feinstein (1991).

3. See, for example, Al-Nowaihi and Pyle (2000).

4. Yaniv (1999) favours the prospect theory instead, and shows that under an obligatory advance tax payments system it is possible to generate a negative relationship between declared income and income tax rate, which would be impossible under an expected utility approach augmented with similar payments scheme.

5. See, among others, Pencavel (1979), Cowell and Gordon (1988), Gordon (1989).

6. It is worth mentioning that various non-constant tax rates have already been introduced in the literature. Thus, Pencavel (1979) considers an endogenous tax schedule but also brings in a labour supply decision and uses alternative penalty functions. Koskela (1983) analyses whether moving to a linear progressive tax schedule increases evasion but provided either expected government revenue or the taxpayer's utility remain unchanged.

7. Note that when the tax liability function is presented as $t(x)+\tau x$, then the marginal tax rate automatically becomes $t'(x)+\tau$. Clearly, a change in τ translates into the change in the marginal tax rate as well. This is important because the puzzle involves an inverse relationship between the marginal tax rate and the amount of evaded income.

8. Observe that there are a number of ways to present the variation in the tax function, but as was pointed out by an anonymous referee, it is simpler and more natural to consider first a tax function in its most general form. I could, for instance, take the tax liability function as θx (as in Yitzhaki's (1974) original paper with θ standing for a constant average and marginal tax rate), and then present the tax rate functions as $(\theta+\tau)x$ and $(\theta+\varepsilon\tau)w$. This would also ensure the claimed result, even though the tax rate is *constant* at x and at w. That is, it is a variation in tax schedule (and not necessarily a specific tax function considered) which drives the main implication of this paper. Further, as was mentioned earlier, some empirical studies find that higher tax rates encourage income declaration. Equation (8'), being ambiguous in sign, allows for such a possibility, too.

9. To further clarify this point, consider a very simple numerical example. Assume that for a given tax policy, a representative individual declares x=90, while his true income, w, is 100. The penalty rate, F, is 1.5. Suppose the total tax liabilities for the income

levels 90 and 100 are equal to 20 and 30, respectively. When caught, the total penalty payment is 15 (=1.5·(30-20)). Now, suppose the government introduces a new tax policy, with higher tax rates than before for all the income levels. As a result, assume the new total tax liabilities become 26 and 35 for the income levels 90 and 100, respectively. Then at x=90 the total penalty payment when caught becomes only 13.5 (=1.5·(35-26)). Since now the evader would make less penalty payments than before (13.5<15), he might reason that declaring x=90 is no longer optimal, and instead decide to declare only, say, 89. Clearly, in this example the amount of income declaration *declined* by 1.

10. Indeed, what happens to the amount of evaded taxes in the previous example? This, of course, would depend on the amount of tax liabilities to be paid under *new* tax policy for the income equal to 89. Previously, the amount of evaded taxes was 10. Now it is 35 minus the amount of taxes evaluated at x=89, whatever the latter is. Formally, in terms of Yitzhaki's (1987) formulation, let $g = t(w)-t(x)$ (taxes evaded) and $c=w-t(w)$ (after-tax true net income), so that $c+g = w-t(x)$. Restate (1) as the maximization of $E[U]=(1-p)U(c+g)+pU(c-\pi g)$ with respect to g, where $\pi=F-1$. The first-order condition is $(1-p)U'(c+g)-p\pi U'(c-\pi g)=0$. It is straightforward to verify that the second-order condition is satisfied if the taxpayer is risk-averse. *Ceteris paribus*, it follows that $\partial g/\partial\tau = (\partial g/\partial c).(\partial c/\partial\tau)$ is zero since a change in the tax rate is assumed to be associated with a compensation to keep the net income constant. But there is no compensation in my setting and moreover, there should be an extra impact captured by the term $[(\partial x/\partial\tau)\,|\,\tau=0].(\partial g/\partial x)$.

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