

Additive Labour Values and Prices: Evidence from the Supply and Use Tables of the French, German and Greek Economies

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ABSTRACT

This paper finds, on the basis of a usual 'square' linear model of joint production, that the vectors of additive labour values and/or actual prices of production associated with the Supply and Use Tables of the Greek economy (for the years 1995 and 1999) are economically insignificant, whilst the relevant vectors of the French (for the years 1995 and 2005) and German (for the years 2000 and 2005) economies are economically significant. The evaluation of the results reveals certain technical-social features of the economies under consideration and casts doubt on the logic of the so-called empirical labour theory of value.

1. INTRODUCTION

CLASSICAL AND MARXIAN VALUE THEORY supports the view that the quantities of labour 'embodied' in the different commodities, or 'labour values', are the main determinants of prices. In recent years there have been a growing number of empirical studies into the relationships between labour values, production prices and market prices. The central conclusion is that, in actual economies, the vectors of labour values and production prices are quite close to that of market prices, as judged by alternative measures of deviation.² Since, however, all these studies are based on Symmetric Input-Output Tables (SIOT), the inclusion of the Supply and Use Tables (SUT) in the exploration seems to be necessary.

As is well known, the SIOT can be derived from the 'System of National Accounts' (SNA) framework of SUT (see, e.g., United Nations, 1999, chs 2-4; Eurostat, 2008, ch. 11), introduced in 1968 (see United Nations, 1968, ch.3).³

Given that in the SUT there are industries that produce more than one commodity and commodities that are produced by more than one industry, it follows that the SUT could be considered as the counterpart of joint production systems à la v. Neumann (1945) and Sraffa (1960, ch. 7).⁴ By contrast, in the SIOT there is neither industry that produces more than one commodity nor commodity that is produced by more than one industry and, therefore, the SIOT could be considered as the *counterpart* of single production systems à la Sraffa (1960, Part 1). Nevertheless, since joint production is the empirically relevant case (see Steedman, 1984; Faber *et al.*, 1998), SUT constitute a *more* realistic 'picture' of the actual economic system than SIOT.

The purpose of this paper is to estimate, in terms of the usual 'square' linear model of production (for a closed economy with circulating capital and homogeneous labour),⁵ the vectors of 'additive labour values' (i.e., direct and indirect labour requirements per unit of net output for each commodity)⁶ and actual prices of production associated with the SUT of three European Union (EU) member states, i.e., France (for the years 1995 and 2005), Germany (for the years 2000 and 2005) and Greece (for the years 1995 and 1999). This data selection is based on the belief that the said economies will possess different production structures: Germany is a large industrial producer; France is the EU's leading agricultural producer; and Greece is a small economy, which is largely based on the tourism and shipping industries (and joined the EU later). Moreover, regarding the Greek economy, there is a relevant empirical study that uses data from the SIOT for the period 1988-1997 (see Tsoulfidis and Mariolis, 2007) and, therefore, a basis for a straightforward comparison of the results derived from SIOT with those derived from SUT.

According to Marx, labour values satisfy: (i) *actuality*, i.e., they are defined by reference to the methods of production actually used in the economy considered; (ii) *additivity*, i.e., the value of the gross product, whether from a single process, an industry or a whole sector of the economy, is equal to the sum of the values of the various means of production used up plus the live labour performed; (iii) *uniqueness*, i.e., they are uniquely determined; and (iv) *positivity*. However, Steedman (1975; 1977, ch. 12) has shown that, when *joint* production is allowed for, the set of properties (i)-(iv) is not necessarily compatible and, more specifically, additive labour values can be non-uniquely determined and/or negative. Furthermore, as is also well known, the prices of production associated with a square, strictly viable and profitable joint production system are not necessarily (semi-) positive (see Sraffa, 1960, §§69-70; Schefold, 1971, Part 1).⁷ Consequently, the possibility that both the additive labour values and the prices of production that correspond to an *actual* system of joint production are economically insignificant cannot be ruled out altogether.⁸ Finally, it need hardly be said that in cases of (i) 'non-square' systems (see Fujimori, 1982, pp. 46-48);⁹ (ii) heterogeneous labour (see Steedman, 1977, ch. 7 and pp. 178-179; 1985); and (iii) non-competitive imports (see Steedman and Metcalfe, 1981, pp. 140-141; Steedman, 2008, pp. 168-173),

any attempt to explore the price-labour value deviation(s) is devoid of economic sense.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the data and the construction of relevant variables. Section 4 subjects the theoretical analysis to empirical testing and evaluates the results of the empirical analysis. Section 5 concludes the paper.

2. THE ANALYTIC FRAMEWORK

Assume a closed capitalist economy, which produces n commodities by n linear processes of pure joint production — i.e., a square, strictly viable and profitable system — and in which commodity prices deviate from the prices of production. Homogeneous labour is the only primary input and there is only circulating capital, whilst labour is not an input to the household sector. Moreover, the net product is distributed to profits and wages that are paid at the beginning of the common production period and there are no savings out of this income.¹⁰ Finally, we assume the following are given (i) the vector of market prices; (ii) the technical conditions of production, that is, the triplet $\{\mathbf{B}, \mathbf{A}, \mathbf{a}\}$, where \mathbf{B} represents the $n \times n$ Make matrix, \mathbf{A} the $n \times n$ Use matrix (both \mathbf{B} and \mathbf{A} are expressed in physical terms), and \mathbf{a}^T the $1 \times n$ vector of employment levels process by process (T is the sign for transpose); and (iii) the real wage rate, which is represented by the $n \times 1$ vector, \mathbf{d} .

On the basis of these assumptions, the vector of additive labour values, \mathbf{v} , the total 'surplus value', S , and the vector of production prices, \mathbf{p} , related to the processes *actually* used in the economy under consideration, may be estimated from the following equations

$$\mathbf{v}^T \mathbf{B} = \mathbf{v}^T \mathbf{A} + \mathbf{a}^T \quad (1)$$

$$S \equiv \mathbf{v}^T \mathbf{u} \quad (2)$$

$$\mathbf{p}^T \mathbf{B} = (1+r)(\mathbf{p}^T \mathbf{A} + w\mathbf{a}^T) \quad (3)$$

or

$$\mathbf{p}^T \mathbf{B} = (1+r)\mathbf{p}^T \mathbf{C} \quad (3a)$$

where $\mathbf{u} \equiv [\mathbf{B} - \mathbf{C}] \mathbf{e}$ represents the 'surplus product', $\mathbf{e} (\equiv [1, 1, \dots, 1]^T)$ the summation vector, $\mathbf{C} (\equiv \mathbf{A} + \mathbf{d}\mathbf{a}^T)$ the 'augmented' Use matrix, $w (= \mathbf{p}^T \mathbf{d})$ the money wage rate in terms of production prices, and r the uniform rate of profits. Provided that $[\mathbf{B} - \mathbf{A}]$ and \mathbf{B} are non-singular, (1), (2) and (3a) entail that

$$\mathbf{v}^T = \mathbf{a}^T [\mathbf{B} - \mathbf{A}]^{-1} \quad (4)$$

$$S = (1 - \mathbf{v}^T \mathbf{d}) \mathbf{a}^T \mathbf{e} \quad (5)$$

$$\mathbf{p}^T = (1 + r) \mathbf{p}^T \mathbf{D} \quad (6)$$

where $\mathbf{D} \equiv \mathbf{C}\mathbf{B}^{-1}$. Equations (4), (5) and (6) imply that: (i) \mathbf{v} is uniquely determined; (ii) S is positive iff the unit ‘value of labour power’, $\mathbf{v}^T \mathbf{d}$, is less than one; and (iii) $(1 + r)^{-1}$ is an eigenvalue of the matrix \mathbf{D} and \mathbf{p}^T is the corresponding left-hand side eigenvector. Nevertheless, *nothing* guarantees the existence of a (semi-) positive solution for $(\mathbf{v}, r, \mathbf{p})$.¹¹

Finally, it should be stressed that any ‘complication’ related to (1)-(2) and/or (3a) (i.e., inconsistency, non-unique solution for \mathbf{v} , non-unique economically significant solution for \mathbf{v} and/or (r, \mathbf{p}) co-existence of positive (non-positive) ‘surplus value’ with non-positive (positive) profits) does not constitute, as is well known, any problem for the v. Neumann/Sraffa-based analysis;¹² it indicates rather an inner limit of the ‘labour theory of value’.

3. DATA AND CONSTRUCTION OF VARIABLES

The SUT and the corresponding levels of sectoral employment of the French (for the years 1995, 1997 and 1999 through 2005) and German (for the years 1995 and 1997 through 2005) economies are available via the Eurostat website (<http://ec.europa.eu/eurostat>), whilst those of the Greek economy (for the years 1995 through 1999) are provided by the National Statistical Service of Greece.

The SUT of the French and German (for the years 2000 through 2005) economies describe 59 products and 59 industries, whilst the SUT of the Greek economy describe 59 products and 60 industries.¹³ The products are classified according to CPA (Classification of Products by Activity), whilst industries are classified according to NACE (General Industrial Classification of Economic Activities within the European Communities). Given that technical change over time could be expected to be rather ‘slow’, we choose to apply our analysis to the tables of (i) the French economy for the years 1995 and 2005; (ii) the German economy for the years 2000 and 2005; and (iii) the Greek economy for the years 1995 and 1999. Thus, setting aside the non-square tables of the German economy (before the year 2000), we maximized the chronological distance amongst the SUT of each country.

In the SUT of the French (for the year 2005), German and Greek economies, all elements associated with the product 12 (Uranium and thorium ores) and industry 12 (Mining of uranium and thorium ores) equal zero and, therefore, we remove them from our analysis. In the case of the French economy, all the elements associated with the ‘primary product’ (Secondary raw materials) of industry 37 (Recycling) are zero and, therefore, we remove

them from our analysis, whilst there are elements associated with the industry 37 that are positive. In order to derive square matrices, we aggregate the industry 37 with the industry 27 (Manufacture of basic metals). This choice is based on the fact that the industry 37 mainly produces the 'primary product' (Basic metals) of the industry 27. Thus, the dimensions of the French Make and Use matrices for the years 1995 and 2005 are 58×58 and 57×57 , respectively. In the case of the German economy, all the elements associated with the product 13 (Metal ores) and industry 13 (Mining of metal ores) of the Make matrices equal zero and, therefore, we remove them from our analysis, whilst there are elements associated with the 'primary product' (Metal ores) of industry 13 in the Use matrices that are positive. In order to derive square matrices, we aggregate the product 13 of the Use matrices with the product 27 (Basic metals). This choice is based on the fact that the product 13 is mainly used by the industry 27. Thus, we derive Make and Use matrices of dimensions 57×57 . Finally, in the case of the Greek economy, the Use matrices include an additional, *fictitious* industry named 'Financial Intermediation Services Indirectly Measured (FISIM)'. In order to derive square matrices, we apply the aggregation that the United Nations (1999, p. 135, §5.76) recommend for this case. Namely, we aggregate the aforesaid industry with industry 65 (Financial intermediation, except insurance and pension funding). Thus, we derive Make and Use matrices of dimensions 58×58 for the Greek economy.

In the Supply Tables, goods and services are measured at current 'basic prices', whilst in the Use Tables all intermediate costs are measured in current 'purchasers' prices'. The derivation of the SUT at basic prices is based on the method proposed by United Nations (1999, ch. 3 and pp. 228-229). Finally, the market prices of all products are taken to be equal to one; that is to say, the physical unit of measurement of each product is that unit which is worth of a monetary unit (see, e.g., Miller and Blair, 1985, p. 356).

Wage differentials are used to homogenize the sectoral employment (see, e.g., Sraffa, 1960, §10; Kurz and Salvadori, 1995, pp. 322-325), i.e., the j th element of the vector of employment levels process by process, $\mathbf{a} \equiv [a_j]$, is determined as follows: $a_j = l_j (w_j^m / w_{\min}^m)$, where l_j and w_j^m are total employment and money wage rate, in terms of market prices, of the j th sector, respectively, and w_{\min}^m is the minimum sectoral money wage rate in terms of market prices. Alternatively, the homogenization of employment could be achieved, for example, through the economy's average wage; in fact, the empirical results are robust to alternative normalizations with respect to homogenization of labour inputs.

Furthermore, by assuming that workers do not save and that their consumption has the same composition as the vector of private households consumption expenditures, \mathbf{c} , directly available in the SUT, the vector of the real wage rate, \mathbf{d} , is determined as follows: $\mathbf{d} = (w_{\min}^m / \mathbf{e}^T \mathbf{c}) \mathbf{c}$ where $\mathbf{e}^T \equiv [1, 1, \dots, 1]$ represents the vector of market prices (see also, e.g., Okishio and Nakatani,

1985, pp. 66-67). It goes without saying that the empirical results (on the deviations of actual prices of production from labour values and market prices) are robust to the assumption that a certain relatively 'small' fraction of wages, s_w , is saved; in this case the vector of the real wage would be equal to $[(1-s_w)w_{\min}^m / \mathbf{e}^T \mathbf{c}] \mathbf{c}$.

Finally, it should be noted that, in the available SUT, we do not have data on fixed capital stocks. As a result, our investigation is based on a model with circulating capital. In the opposite case equation (3) becomes

$$\mathbf{p}^T \mathbf{B}^+ = (1+r)(\mathbf{p}^T \mathbf{A} + w \mathbf{a}^T) \quad (7)$$

where matrix \mathbf{B}^+ allows both for pure joint products and utilized fixed capital goods (approach). If \mathbf{B}^+ could be replaced by $\mathbf{B} + \mathbf{A}^F - \mathbf{A}^D$, where \mathbf{A}^F denotes the matrix of capital stock coefficients and \mathbf{A}^D denotes the matrix of depreciation coefficients (Leontief-Bródy approach),¹⁴ then (7) should be written as

$$\mathbf{p}^T [\mathbf{B} + \mathbf{A}^F - \mathbf{A}^D] = \mathbf{p}^T \mathbf{A} + (1+r) (\mathbf{p}^T \mathbf{A}^F + w \mathbf{a}^T)$$

or

$$\mathbf{p}^T \mathbf{B} = \mathbf{p}^T [\mathbf{A} + \mathbf{A}^D] + r \mathbf{p}^T \mathbf{A}^F + (1+r) w \mathbf{a}^T \quad (7a)$$

Equation (7a), without pure joint products (i.e., \mathbf{B} is a diagonal matrix), is used, more often than not, for the analysis of price-labour value deviations in actual economies (see, e.g., Sekerka *et al.*, 1970; Mathur, 1977; Ochoa 1989).

4. RESULTS AND THEIR EVALUATION

The application of our analysis to the SUT of the French, German and Greek economies gives the following results:¹⁵

(i). The matrices $[\mathbf{B} - \mathbf{A}]$ and \mathbf{B} are non-singular. Consequently, \mathbf{v} can be uniquely estimated from (4), and \mathbf{p} can be estimated from (6).

(ii). The matrices $[\mathbf{B} - \mathbf{A}]^{-1}$ contain negative elements. Consequently, the systems under consideration are not 'all-productive' and, therefore, they do not have the properties of a single-product system (Schefold, 1971, pp. 34-35; 1978; see also Kurz and Salvadori, 1995, pp. 238-240).¹⁶

(iii). The vectors of labour values of the French, German and the Greek (for the year 1995) economies are positive. However, the vector of labour values of the Greek economy for the year 1999 contains five *negative* elements, which correspond to the 'primary products' of the following industries: 01 (Agriculture, hunting and related service activities); 11 (Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying); 23 (Manufacture of coke, refined petroleum products and nuclear

fuels); 61 (Water transport); and 67 (Activities auxiliary to financial intermediation).¹⁷ As is well known, some labour values are negative iff a non-negative linear combination of some industries yields a greater net output per unit of labour employed than a non-negative linear combination of the remaining ones (see Filippini and Filippini, 1982, pp. 387-388).

(iv). The 'surplus values' are positive (French economy: $\mathbf{v}^T \mathbf{d} \approx 0.48$ (1995), 0.48 (2005), German economy: $\mathbf{v}^T \mathbf{d} \approx 0.48$ (2000), 0.45 (2005), Greek economy: $\mathbf{v}^T \mathbf{d} \approx 0.28$ (1995), 0.16 (1999)).

(v). The systems of production prices of the French and German economies have a unique, positive solution for (r, \mathbf{p}) , and $(1+r)^{-1}$ are the *dominant* eigenvalues of the matrices \mathbf{D} . Thus, it is found that the actual uniform rates of profits of the French economy are approximately 33 per cent (1995) and 32 per cent (2005), whilst those of the German economy are about 35 per cent (2000) and 38 per cent (2005).¹⁸ The system of production prices of the Greek economy for the year 1995 has 20 positive, 4 negative and 34 complex conjugate solutions for r , whilst that for the year 1999 has 16 positive, 8 negative and 34 complex conjugate solutions, respectively. However, all the corresponding solutions for \mathbf{p} are economically insignificant (i.e., non-semi-positive). Consequently, in the case of the Greek economy, positive 'surplus value' co-exists with economically insignificant (r, \mathbf{p}) .¹⁹

(vi). In the French economy, the deviations of actual prices of production from additive labour values are around 15 per cent, whilst the deviations of market prices from additive labour values and actual prices of production for the year 1995 are in the range of 88-90 per cent and 44-45 per cent for 2005.²⁰ In the German economy, the deviations of actual prices of production from additive labour values are in the range of 14-16 per cent, whilst the deviations of market prices from additive labour values and actual prices of production are in the range of 56-58 per cent.²¹ Finally, in the Greek economy for the year 1995, the deviation of market prices from additive labour values is almost 87 percent (see Table 1).

Table 1. Deviations of prices from additive labour values; French, German and Greek economies

| d - distance (%) | <i>Actual prices of production vs. additive labour values</i> | <i>Market prices vs. additive labour values</i> | <i>Market prices vs. actual prices of production</i> |
|--------------------|---|---|--|
| France 1995 | 14.5 | 87.8 | 89.8 |
| France 2005 | 15.0 | 44.3 | 44.8 |
| Germany 2000 | 14.1 | 56.8 | 57.7 |
| Germany 2005 | 16.1 | 56.3 | 57.5 |
| Greece 1995 | - | 87.0 | - |
| Greece 1999 | - | - | - |

Thus, we conclude that the deviations of market prices from additive labour values and actual prices of production are, by and large, considerably greater than those estimated on the basis of SIOT.²² For example, the market price-labour value deviation estimated on the basis of the 19 x19 SIOT of the Greek economy for the year 1995 is almost 23.6 per cent (see Tsoulfidis and Mariolis, 2007, p. 428, Table 1). Since the *theoretically* maximum value of $\cos\theta$ equals $1/\sqrt{n}$, the theoretically maximum value of the ‘*d* - distance’, D , equals $\sqrt{2[1-(1/\sqrt{n})]}$. Thus, the normalized ‘*d* - distance’, defined as d/D , is almost 23.6/124.2 or 19 per cent and, therefore, considerably lower than the normalized ‘*d* - distance’ estimated from the relevant 58 x 58 SUT, which is almost 87.0/131.8 or 66 per cent.

The next issue that comes up is whether the systems under consideration are ‘*r*-all-engaging’, i.e., characterized by $\mathbf{E}(r) \equiv [\mathbf{B} - (1+r)\mathbf{A}]^{-1} > \mathbf{0}$ for some $r > -1$.²³ As is well known, $\mathbf{E}(r) > \mathbf{0}$ is a sufficient condition for the existence of an *interval* of r , in which a joint production system retains all the essential properties of indecomposable single-product systems (see Schefold, 1971, p. 35; 1978; Bidard, 1996).²⁴ The investigation can be based on the following theorem (Bidard, 1996, p. 328): Consider the eigensystems associated with the pair $\{\mathbf{B}, \mathbf{A}\}$, namely

$$\lambda \mathbf{B}\mathbf{x} = \mathbf{A}\mathbf{x} \tag{8}$$

$$\lambda \mathbf{y}^T \mathbf{B} = \mathbf{y}^T \mathbf{A} \tag{9}$$

The system $\{\mathbf{B}, \mathbf{A}\}$ is ‘*r*-all-engaging’ iff there exist $(\lambda, \mathbf{x}, \mathbf{y}) > \mathbf{0}$, where \mathbf{x} is determined up to a factor.²⁵

The estimation of the characteristic values and vectors associated with the pairs $\{\mathbf{B}, \mathbf{A}\}$ of the French, German and Greek economies gives the following results: (i) the eigensystems of the French economy have 21 positive (and simple) eigenvalues for each year of our analysis; (ii) the eigensystems of the German economy for the years 2000 and 2005 have 21 positive (and simple) and 17 positive (and simple) eigenvalues, respectively; and (iii) the eigensystems of the Greek economy for the years 1995 and 1999 have 18 positive (and simple) and 15 positive (and simple) eigenvalues, respectively.²⁶ In the case of the French economy, it is found that the dominant eigenvalues and the corresponding right and left eigenvectors are positive, which implies that $\mathbf{E}(r) > \mathbf{0}$ for $r_0 \leq r \leq R \equiv (\lambda^*)^{-1} - 1$ (see Bidard, 1996, p. 329), where $r_0 \approx 87$ per cent (78 per cent) for the year 1995 (2005), λ^* is the dominant eigenvalue and $R \approx 96$ per cent (1995), 90 per cent (2005). Thus, although the French economy constitutes an ‘*r* -all-engaging system’, the *actual* uniform rates of profits of the economy (i.e., 33 per cent (1995) and 32 per cent (2005)) do not belong to the interval $[r_0, R]$. Finally, in the case of the German and Greek economies, there are no positive eigenvectors and, therefore, they are not ‘*r* -all-engaging’.

5. CONCLUDING REMARKS

The exploration of the relationships between additive labour values and actual prices using a usual linear model of joint production and data from the Supply and Use Tables of the French, German and Greek economies, gave the following results:

(i). The systems under consideration are not 'all-productive' and, therefore, they do not have the properties of a single-product system.

(ii). In the French (for the years 1995 and 2005) and German (for the years 2000 and 2005) economies, positive additive labour values and positive actual prices of production co-exist with positive 'surplus value'. Also, the actual uniform rates of profits are in the range of 32-33 per cent and 35-38 per cent, respectively. By contrast, in the Greek economy (for the years 1995 and 1999), economically insignificant additive labour values and/or actual prices of production co-exist with positive 'surplus value'.

(iii). Although the deviations of actual prices of production from additive labour values are in the range of 14-16 per cent (as this can be judged from the '*d* - distance'), the deviations of market prices from additive labour values and actual prices of production are in the range of 44-90 per cent.

(iv). In the French economy for the year 1995 (2005), there is an interval of the uniform rate of profits, i.e., 87-96 per cent (78-90 per cent), in which the economy is '*r*-all-engaging' and, therefore, behaves as indecomposable single-product systems. However, the said interval does not include the actual uniform rate of profits of the economy. By contrast, the German and Greek economies are not '*r*-all-engaging'.

Since in the real world joint production constitutes the rule, these findings would seem to be of some importance: they reveal certain technical-social features of the economies under consideration and, at the same time, cast doubt on the logic of the 'empirical labour theory of value' (Stigler, 1958, p. 361). Nevertheless, future research efforts should use more disaggregated input-output data from various countries, concretize the model by including the presence of fixed capital and the degree of its utilization, depreciation, turnover times, taxes and subsidies, and explore the relationships between prices and hypothetical changes in income distribution.

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APPENDIX

1. Additive labour values (ALV) of the French, German and Greek economies

Table 1.1. ALV; France, 1995

| CPA | ALV | CPA | ALV |
|---------|-------|-----|-------|
| 01 | 54.6 | 36 | 113.6 |
| 02 | 87.3 | 40 | 79.9 |
| 05 | 67.9 | 41 | 120.2 |
| 10 | 323.7 | 45 | 106.0 |
| 11 | 142.7 | 50 | 93.1 |
| 12 | 15.2 | 51 | 109.0 |
| 13 | 8.2 | 52 | 103.5 |
| 14 | 106.4 | 55 | 103.5 |
| 15 | 86.5 | 60 | 103.7 |
| 16 | 80.8 | 61 | 112.0 |
| 17 | 107.3 | 62 | 123.8 |
| 18 | 108.9 | 63 | 123.1 |
| 19 | 101.8 | 64 | 110.5 |
| 20 | 113.0 | 65 | 107.4 |
| 21 | 101.4 | 66 | 107.4 |
| 22 | 117.9 | 67 | 121.5 |
| 23 | 98.1 | 70 | 24.4 |
| 24 | 95.5 | 71 | 42.1 |
| 25 | 109.0 | 72 | 154.6 |
| 26 | 111.5 | 73 | 120.7 |
| 27 ⊕ 37 | 104.6 | 74 | 110.8 |
| 28 | 112.4 | 75 | 135.6 |
| 29 | 125.4 | 80 | 155.1 |
| 30 | 83.9 | 85 | 114.8 |
| 31 | 110.7 | 90 | 78.2 |
| 32 | 136.6 | 91 | 108.9 |
| 33 | 103.8 | 92 | 100.6 |
| 34 | 106.5 | 93 | 95.3 |
| 35 | 132.6 | 95 | 133.8 |

Table 1.2. ALV; France, 2005

| CPA | ALV | CPA | ALV |
|---------|-------|-----|-------|
| 01 | 41.2 | 40 | 46.9 |
| 02 | 35.8 | 41 | 56.9 |
| 05 | 37.7 | 45 | 64.0 |
| 10 | 132.1 | 50 | 64.4 |
| 11 | 38.4 | 51 | 69.5 |
| 13 | 139.6 | 52 | 63.6 |
| 14 | 63.3 | 55 | 64.8 |
| 15 | 58.3 | 60 | 66.2 |
| 16 | 42.3 | 61 | 52.4 |
| 17 | 70.7 | 62 | 69.1 |
| 18 | 63.5 | 63 | 65.4 |
| 19 | 70.2 | 64 | 59.4 |
| 20 | 64.5 | 65 | 70.1 |
| 21 | 65.6 | 66 | 59.6 |
| 22 | 70.5 | 67 | 62.4 |
| 23 | 42.0 | 70 | 14.6 |
| 24 | 60.2 | 71 | 38.4 |
| 25 | 69.2 | 72 | 75.2 |
| 26 | 66.4 | 73 | 84.7 |
| 27 ⊕ 37 | 67.0 | 74 | 71.9 |
| 28 | 74.4 | 75 | 85.0 |
| 29 | 74.4 | 80 | 94.9 |
| 30 | 59.5 | 85 | 73.8 |
| 31 | 76.3 | 90 | 43.1 |
| 32 | 73.7 | 91 | 69.0 |
| 33 | 74.6 | 92 | 63.7 |
| 34 | 67.8 | 93 | 49.8 |
| 35 | 71.0 | 95 | 112.8 |
| 36 | 68.9 | | |

Table 1.3. ALV; Germany, 2000

| CPA | ALV | CPA | ALV |
|---------|-------|-----|-------|
| 01 | 46.7 | 40 | 63.4 |
| 02 | 57.0 | 41 | 34.0 |
| 05 | 39.7 | 45 | 64.6 |
| 10 | 172.5 | 50 | 71.7 |
| 11 | 38.1 | 51 | 62.5 |
| 14 | 55.5 | 52 | 66.6 |
| 15 | 58.3 | 55 | 61.3 |
| 16 | 42.7 | 60 | 75.9 |
| 17 | 68.6 | 61 | 38.4 |
| 18 | 66.5 | 62 | 50.2 |
| 19 | 68.8 | 63 | 63.7 |
| 20 | 63.2 | 64 | 47.2 |
| 21 | 63.4 | 65 | 64.3 |
| 22 | 55.7 | 66 | 61.4 |
| 23 | 37.6 | 67 | 43.6 |
| 24 | 63.7 | 70 | 17.4 |
| 25 | 68.5 | 71 | 10.7 |
| 26 | 65.9 | 72 | 64.9 |
| 27 ⊕ 13 | 67.4 | 73 | 72.6 |
| 28 | 71.2 | 74 | 50.8 |
| 29 | 73.8 | 75 | 82.6 |
| 30 | 59.5 | 80 | 94.6 |
| 31 | 73.3 | 85 | 68.9 |
| 32 | 66.4 | 90 | 47.1 |
| 33 | 73.0 | 91 | 95.6 |
| 34 | 74.5 | 92 | 49.1 |
| 35 | 78.0 | 93 | 26.4 |
| 36 | 70.6 | 95 | 110.8 |
| 37 | 56.5 | | |

Table 1.4. ALV; Germany, 2005

| CPA | ALV | CPA | ALV |
|---------|-------|-----|-------|
| 01 | 52.3 | 40 | 52.0 |
| 02 | 39.4 | 41 | 33.1 |
| 05 | 37.0 | 45 | 63.6 |
| 10 | 160.1 | 50 | 68.4 |
| 11 | 47.9 | 51 | 64.2 |
| 14 | 63.4 | 52 | 68.3 |
| 15 | 60.2 | 55 | 64.4 |
| 16 | 49.1 | 60 | 77.2 |
| 17 | 67.1 | 61 | 36.8 |
| 18 | 65.1 | 62 | 55.3 |
| 19 | 62.9 | 63 | 61.0 |
| 20 | 59.6 | 64 | 42.9 |
| 21 | 61.8 | 65 | 56.1 |
| 22 | 55.1 | 66 | 57.9 |
| 23 | 45.3 | 67 | 40.2 |
| 24 | 60.3 | 70 | 15.1 |
| 25 | 64.9 | 71 | 11.4 |
| 26 | 68.2 | 72 | 74.5 |
| 27 ⊕ 13 | 61.5 | 73 | 80.4 |
| 28 | 69.6 | 74 | 54.2 |
| 29 | 71.8 | 75 | 84.4 |
| 30 | 63.4 | 80 | 96.5 |
| 31 | 77.4 | 85 | 68.9 |
| 32 | 66.1 | 90 | 46.9 |
| 33 | 68.4 | 91 | 96.6 |
| 34 | 71.3 | 92 | 52.0 |
| 35 | 74.4 | 93 | 26.6 |
| 36 | 68.1 | 95 | 116.4 |
| 37 | 59.1 | | |

Table 1.5. ALV; Greece, 1995

| <i>CPA</i> | <i>ALV</i> | <i>CPA</i> | <i>ALV</i> |
|------------|------------|------------|------------|
| 01 | 148.8 | 37 | 238.9 |
| 02 | 1244.4 | 40 | 482.9 |
| 05 | 356.8 | 41 | 674.3 |
| 10 | 903.5 | 45 | 460.2 |
| 11 | 195.1 | 50 | 301.1 |
| 13 | 967.3 | 51 | 413.8 |
| 14 | 482.3 | 52 | 220.3 |
| 15 | 339.3 | 55 | 261.2 |
| 16 | 290.5 | 60 | 488.0 |
| 17 | 505.5 | 61 | 486.2 |
| 18 | 479.0 | 62 | 731.7 |
| 19 | 482.5 | 63 | 845.6 |
| 20 | 523.4 | 64 | 541.6 |
| 21 | 575.8 | 65 | 1844.8 |
| 22 | 652.6 | 66 | 678.9 |
| 23 | 280.5 | 67 | 297.4 |
| 24 | 614.5 | 70 | 39.5 |
| 25 | 605.4 | 71 | 194.2 |
| 26 | 646.0 | 72 | 594.5 |
| 27 | 546.1 | 73 | 693.2 |
| 28 | 623.6 | 74 | 385.7 |
| 29 | 682.9 | 75 | 1027.8 |
| 30 | 580.9 | 80 | 994.6 |
| 31 | 620.2 | 85 | 538.4 |
| 32 | 586.4 | 90 | 742.4 |
| 33 | 666.6 | 91 | 899.5 |
| 34 | 585.2 | 92 | 557.9 |
| 35 | 1009.9 | 93 | 231.5 |
| 36 | 486.2 | 95 | 1308.3 |

Table 1.6. ALV; Greece, 1999

| <i>CPA</i> | <i>ALV</i> | <i>CPA</i> | <i>ALV</i> |
|------------|------------|------------|------------|
| 01 | -27.3 | 37 | 136.0 |
| 02 | 1651.7 | 40 | 32.8 |
| 05 | 31.0 | 41 | 895.2 |
| 10 | 637.9 | 45 | 313.7 |
| 11 | -7852.7 | 50 | 291.9 |
| 13 | 814.8 | 51 | 329.4 |
| 14 | 169.9 | 52 | 163.0 |
| 15 | 201.1 | 55 | 169.9 |
| 16 | 240.6 | 60 | 173.8 |
| 17 | 441.3 | 61 | -270.9 |
| 18 | 428.3 | 62 | 397.1 |
| 19 | 431.9 | 63 | 618.5 |
| 20 | 467.2 | 64 | 318.5 |
| 21 | 485.2 | 65 | 1459.5 |
| 22 | 585.9 | 66 | 525.1 |
| 23 | -5857.4 | 67 | -14.6 |
| 24 | 451.9 | 70 | 29.7 |
| 25 | 501.7 | 71 | 145.2 |
| 26 | 277.0 | 72 | 688.1 |
| 27 | 372.2 | 73 | 677.8 |
| 28 | 499.8 | 74 | 399.5 |
| 29 | 631.5 | 75 | 971.3 |
| 30 | 666.3 | 80 | 1044.3 |
| 31 | 491.1 | 85 | 542.2 |
| 32 | 494.8 | 90 | 573.8 |
| 33 | 596.0 | 91 | 844.3 |
| 34 | 450.3 | 92 | 493.9 |
| 35 | 979.6 | 93 | 188.5 |
| 36 | 468.8 | 95 | 1280.6 |

2. Eigenvalues of the systems of production prices of the French, German and Greek economies

Table 2.1 Eigenvalues of **D**; France, 1995

| | |
|--------------------|------------------------|
| 0.750 | $0.088 \pm 0.015i$ |
| 0.416 | $0.077 \pm 0.002i$ |
| 0.365 | $0.030 \pm 0.057i$ |
| 0.354 | $0.060 \pm 0.007i$ |
| 0.319 | $0.057 \pm 0.001i$ |
| 0.283 | $0.016 \pm 0.048i$ |
| $0.243 \pm 0.064i$ | 0.046 |
| 0.251 | -0.043 |
| 0.239 | 0.039 |
| 0.228 | $0.037 \pm 0.010i$ |
| 0.225 | $0.007 \pm 0.026i$ |
| $0.210 \pm 0.004i$ | -0.024 |
| 0.169 | 0.019 |
| $0.156 \pm 0.032i$ | 0.013 |
| 0.135 | $0.009 \pm 0.002i$ |
| $0.115 \pm 0.051i$ | 0.004 |
| 0.113 | $-0.002 \pm 0.002i$ |
| $0.103 \pm 0.005i$ | $0.001 \pm 0.001i$ |
| 0.098 | -0.00005 |
| $0.095 \pm 0.007i$ | 1.082×10^{-6} |
| 0.093 | |

Table 2.2 Eigenvalues of **D**; France, 2005

| | |
|--------------------|---------------------|
| 0.757 | $0.098 \pm 0.009i$ |
| 0.411 | 0.087 |
| 0.361 | $0.051 \pm 0.069i$ |
| $0.246 \pm 0.064i$ | $0.081 \pm 0.002i$ |
| $0.251 \pm 0.003i$ | 0.069 |
| $0.251 \pm 0.020i$ | 0.064 |
| 0.247 | $0.008 \pm 0.063i$ |
| $0.225 \pm 0.012i$ | $0.053 \pm 0.002i$ |
| 0.201 | $-0.049 \pm 0.001i$ |
| 0.193 | $0.045 \pm 0.018i$ |
| $0.175 \pm 0.033i$ | 0.042 |
| 0.163 | $0.023 \pm 0.002i$ |
| 0.159 | $0.019 \pm 0.012i$ |
| $0.152 \pm 0.038i$ | 0.011 |
| 0.145 | 0.005 |
| $0.133 \pm 0.032i$ | $0.002 \pm 0.003i$ |
| $0.121 \pm 0.063i$ | -0.004 |
| 0.118 | -0.001 |
| 0.108 | -0.0001 |
| 0.102 | |

Table 2.3 Eigenvalues of **D**; Germany, 2000

| | |
|--------------------|---------------------|
| 0.740 | 0.091 |
| 0.358 | $-0.050 \pm 0.069i$ |
| 0.316 | 0.083 |
| $0.297 \pm 0.026i$ | $0.076 \pm 0.014i$ |
| 0.288 | -0.072 |
| 0.256 | 0.069 |
| 0.246 | $0.058 \pm 0.028i$ |
| $0.241 \pm 0.006i$ | 0.061 |
| $0.216 \pm 0.015i$ | 0.059 |
| 0.207 | $0.032 \pm 0.045i$ |
| 0.195 | $-0.032 \pm 0.039i$ |
| $0.172 \pm 0.071i$ | 0.034 |
| 0.185 | 0.032 |
| $0.183 \pm 0.007i$ | 0.019 |
| $0.173 \pm 0.042i$ | $0.016 \pm 0.003i$ |
| 0.158 | $0.001 \pm 0.015i$ |
| 0.145 | $-0.003 \pm 0.006i$ |
| $0.136 \pm 0.007i$ | 0.002 |
| $0.119 \pm 0.005i$ | 0.0001 |
| $0.098 \pm 0.033i$ | |
| 0.096 | |

Table 2.4 Eigenvalues of **D**; Germany, 2005

| | |
|---------------------|---------------------|
| 0.727 | 0.097 |
| 0.383 | $0.094 \pm 0.018i$ |
| 0.316 | 0.095 |
| $0.297 \pm 0.006i$ | 0.080 |
| 0.290 | 0.069 |
| 0.257 | 0.066 |
| 0.244 | 0.059 |
| $0.230 \pm 0.004i$ | -0.059 |
| $0.225 \pm 0.012i$ | $0.021 \pm 0.044i$ |
| 0.221 | $0.046 \pm 0.010i$ |
| $0.182 \pm 0.071i$ | -0.044 |
| 0.190 | 0.043 |
| $0.183 \pm 0.044i$ | $0.026 \pm 0.008i$ |
| 0.186 | $-0.004 \pm 0.024i$ |
| 0.169 | -0.020 |
| $0.154 \pm 0.009i$ | 0.019 |
| $0.141 \pm 0.025i$ | 0.015 |
| 0.141 | $0.002 \pm 0.012i$ |
| $0.122 \pm 0.010i$ | 0.0003 |
| $0.114 \pm 0.035i$ | 0.0001 |
| $-0.034 \pm 0.091i$ | |

Table 2.5 Eigenvalues of **D**; Greece, 1995

| | |
|---------------------|--------------------------|
| 0.711 | $0.009 \pm 0.051i$ |
| 0.540 | $-0.036 \pm 0.032i$ |
| 0.370 | 0.047 |
| 0.350 | $-0.041 \pm 0.017i$ |
| 0.281 | 0.041 |
| 0.257 | $0.011 \pm 0.023i$ |
| 0.196 | 0.024 |
| $0.191 \pm 0.016i$ | $0.018 \pm 0.005i$ |
| $0.132 \pm 0.131i$ | $-0.014 \pm 0.006i$ |
| 0.162 | $-0.006 \pm 0.012i$ |
| $0.136 \pm 0.009i$ | 0.011 |
| 0.126 | -0.008 |
| $-0.077 \pm 0.091i$ | 0.008 |
| 0.100 | 0.007 |
| $0.029 \pm 0.073i$ | $-0.002 \pm 0.002i$ |
| $0.066 \pm 0.041i$ | 0.002 |
| $0.070 \pm 0.016i$ | -0.001 |
| 0.071 | 0.0003 |
| -0.067 | 1.061×10^{-7} |
| $-0.013 \pm 0.062i$ | -3.418×10^{-19} |
| $0.056 \pm 0.002i$ | |

Table 2.6 Eigenvalues of **D**; Greece, 1999

| | |
|---------------------|-------------------------|
| $0.461 \pm 0.511i$ | $-0.009 \pm 0.044i$ |
| 0.645 | -0.041 |
| 0.403 | $0.002 \pm 0.039i$ |
| 0.344 | $-0.031 \pm 0.019i$ |
| 0.256 | $0.029 \pm 0.022i$ |
| $-0.209 \pm 0.140i$ | 0.034 |
| $0.200 \pm 0.125i$ | -0.022 |
| $0.189 \pm 0.035i$ | $-0.005 \pm 0.016i$ |
| 0.185 | $0.015 \pm 0.005i$ |
| 0.165 | $0.003 \pm 0.011i$ |
| 0.153 | -0.011 |
| 0.121 | -0.010 |
| 0.110 | 0.008 |
| $0.107 \pm 0.025i$ | $-0.002 \pm 0.002i$ |
| 0.104 | 0.002 |
| $0.079 \pm 0.019i$ | -0.001 |
| $0.056 \pm 0.029i$ | 0.001 |
| -0.059 | -0.0002 |
| $0.056 \pm 0.012i$ | -1.018×10^{-7} |
| 0.050 | 1.411×10^{-18} |
| $0.019 \pm 0.043i$ | |

3. Eigenvalues of the pairs **{B, A}** of the French, German and Greek economies

Table 3.1 Eigenvalues of **{B, A}**; France, 1995

| | |
|--------------------|-------------------------|
| 0.509 | 0.073 |
| 0.416 | $0.031 \pm 0.061i$ |
| 0.362 | $0.063 \pm 0.006i$ |
| 0.351 | 0.059 |
| $0.294 \pm 0.012i$ | 0.054 |
| $0.250 \pm 0.032i$ | 0.051 |
| 0.251 | $0.035 \pm 0.023i$ |
| 0.239 | 0.039 |
| 0.229 | -0.035 |
| 0.226 | $0.034 \pm 0.006i$ |
| $0.207 \pm 0.007i$ | $-0.022 \pm 0.017i$ |
| $0.178 \pm 0.027i$ | 0.020 |
| 0.159 | 0.014 |
| $0.114 \pm 0.052i$ | $0.009 \pm 0.002i$ |
| 0.117 | -0.005 |
| 0.110 | 0.004 |
| $0.102 \pm 0.004i$ | 0.003 |
| 0.098 | -0.0004 |
| $0.096 \pm 0.003i$ | $-0.00002 \pm 0.00003i$ |
| $0.089 \pm 0.014i$ | 0 |
| $0.080 \pm 0.003i$ | 0 |

Table 3.2 Eigenvalues of **{B, A}**; France, 2005

| | |
|--------------------|---------------------|
| 0.525 | 0.093 |
| 0.408 | $0.051 \pm 0.073i$ |
| 0.356 | 0.085 |
| $0.252 \pm 0.042i$ | $0.083 \pm 0.002i$ |
| $0.254 \pm 0.012i$ | 0.069 |
| 0.249 | $0.063 \pm 0.002i$ |
| 0.240 | 0.050 |
| 0.236 | $-0.048 \pm 0.007i$ |
| $0.234 \pm 0.013i$ | $0.043 \pm 0.023i$ |
| $0.198 \pm 0.034i$ | 0.043 |
| 0.193 | $0.022 \pm 0.020i$ |
| $0.164 \pm 0.028i$ | $0.023 \pm 0.006i$ |
| 0.163 | $0.011 \pm 0.010i$ |
| $0.161 \pm 0.005i$ | 0.012 |
| 0.145 | -0.008 |
| $0.113 \pm 0.063i$ | 0.005 |
| $0.124 \pm 0.025i$ | 0.003 |
| 0.116 | -0.0001 |
| 0.108 | 0 |
| 0.103 | 0 |
| 0.101 | |

Table 3.3 Eigenvalues of $\{B, A\}$; Germany, 2000 Table 3.4 Eigenvalues of $\{B, A\}$; Germany, 2005

| | | | |
|--------------------|---------------------|---------------------|---------------------|
| 0.488 | 0.089 | 0.502 | $0.095 \pm 0.001i$ |
| 0.359 | $-0.045 \pm 0.071i$ | 0.383 | $0.085 \pm 0.026i$ |
| 0.314 | $0.076 \pm 0.005i$ | 0.318 | 0.069 |
| $0.295 \pm 0.024i$ | -0.071 | $0.296 \pm 0.007i$ | $0.064 \pm 0.001i$ |
| 0.286 | $0.058 \pm 0.039i$ | $0.263 \pm 0.025i$ | -0.057 |
| $0.246 \pm 0.009i$ | 0.068 | 0.244 | 0.051 |
| 0.242 | 0.064 | $0.230 \pm 0.008i$ | $0.047 \pm 0.019i$ |
| 0.238 | 0.059 | $0.224 \pm 0.010i$ | -0.047 |
| $0.215 \pm 0.025i$ | 0.053 | 0.220 | 0.047 |
| 0.208 | $-0.034 \pm 0.022i$ | $0.197 \pm 0.057i$ | 0.025 |
| $0.201 \pm 0.010i$ | 0.035 | $0.195 \pm 0.030i$ | $0.019 \pm 0.010i$ |
| 0.183 | $0.027 \pm 0.008i$ | 0.186 | -0.021 |
| $0.181 \pm 0.014i$ | 0.024 | 0.176 | 0.020 |
| $0.167 \pm 0.046i$ | 0.017 | 0.168 | $0.003 \pm 0.015i$ |
| 0.147 | 0.015 | $0.153 \pm 0.006i$ | 0.015 |
| $0.144 \pm 0.014i$ | $-0.003 \pm 0.012i$ | 0.138 | $-0.004 \pm 0.008i$ |
| $0.139 \pm 0.007i$ | $0.004 \pm 0.009i$ | $0.122 \pm 0.033i$ | 0.0001 |
| 0.132 | 0.0001 | $0.115 \pm 0.020i$ | 0 |
| 0.112 | 0 | $0.116 \pm 0.010i$ | |
| $0.104 \pm 0.028i$ | | 0.111 | |
| $0.092 \pm 0.008i$ | | $-0.031 \pm 0.090i$ | |

Table 3.5 Eigenvalues of $\{B, A\}$; Greece, 1995 Table 3.6 Eigenvalues of $\{B, A\}$; Greece, 1999

| | | | |
|---------------------|-----------------------|---------------------|---------------------|
| 0.689 | 0.044 | $0.397 \pm 0.515i$ | $-0.008 \pm 0.041i$ |
| 0.420 | 0.043 | 0.648 | -0.042 |
| 0.363 | $0.031 \pm 0.027i$ | 0.400 | $0.023 \pm 0.027i$ |
| 0.348 | $-0.037 \pm 0.019i$ | 0.337 | $0.033 \pm 0.007i$ |
| 0.282 | $0.006 \pm 0.040i$ | 0.255 | 0.028 |
| 0.250 | -0.039 | $-0.208 \pm 0.140i$ | $-0.021 \pm 0.018i$ |
| 0.199 | $-0.016 \pm 0.023i$ | $0.207 \pm 0.047i$ | $-0.006 \pm 0.022i$ |
| 0.194 | 0.025 | 0.185 | -0.019 |
| $0.144 \pm 0.108i$ | $0.022 \pm 0.008i$ | $0.181 \pm 0.026i$ | $-0.013 \pm 0.010i$ |
| $0.161 \pm 0.010i$ | $-0.003 \pm 0.018i$ | $0.147 \pm 0.013i$ | $0.015 \pm 0.005i$ |
| $0.129 \pm 0.009i$ | -0.014 | 0.126 | $-0.003 \pm 0.013i$ |
| 0.128 | 0.008 | 0.121 | 0.008 |
| $-0.076 \pm 0.093i$ | -0.007 | 0.112 | -0.007 |
| 0.115 | 0.006 | 0.101 | 0.003 |
| $0.029 \pm 0.073i$ | $0.003 \pm 0.001i$ | 0.081 | $0.001 \pm 0.001i$ |
| $0.066 \pm 0.036i$ | -0.003 | $0.067 \pm 0.033i$ | -0.001 |
| -0.069 | 0.001 | 0.067 | -0.0001 |
| $0.066 \pm 0.003i$ | $-0.0001 \pm 0.0005i$ | $0.034 \pm 0.049i$ | 0 |
| $-0.010 \pm 0.061i$ | 0 | -0.060 | 0 |
| 0.060 | 0 | $0.046 \pm 0.021i$ | 0 |
| 0.051 | 0 | 0.050 | |

ENDNOTES

1. Department of Public Administration, Panteion University, 136 Syngrou Ave, 17671 Athens, Greece; E-mail: mariolis@hotmail.gr (corresponding author) and gsok@hotmail.gr, respectively. Earlier versions of this paper were presented at a Workshop in Political Economy at the Panteion University, in March 2007, and at a Workshop of the 'Study Group on Sraffian Economics' at the Panteion University, in January 2008: We are indebted to Sobei H. Oda, Eleftheria Rodousaki, Nikolaos Rodousakis and Lefteris Tsoulfidis for helpful discussions and comments. We are also grateful to Nikolaos Stropoulos (Director of the National Accounts Division of the National Statistical Service of Greece) for his kind advice concerning the Supply and Use Tables of the Greek economy. Finally, we would like to express our appreciation to two referees for extremely helpful comments and suggestions. The usual disclaimer applies.

2. See Shaikh (1984, 1998), Petrovic (1987), Ochoa (1989), Cockshott et al. (1995), Cockshott and Cottrell (1997), Chilcote (1997), Tsoulfidis and Maniatis (2002), Zachariah (2006), Tsoulfidis (2008) *inter alia*. A remarkable exception can be found in Steedman and Tomkins (1998), where the production price-labour value deviations are greater than those usually estimated.

3. For a review of the methods, used to convert the SUT into SIOT, see, e.g., ten Raa and Rueda-Cantuche (2003, pp. 441-447). Amongst the various available methods, the so-called 'Commodity Technology Assumption' is the only one that fulfils a set of important properties of the input-output analysis (see Jansen and ten Raa, 1990). However, the 'Commodity Technology Assumption' is possible to generate economically insignificant results, i.e., *negative* elements in the input-output matrix. For a critical review of the various procedures proposed to overcome this inconsistency, see ten Raa and Rueda-Cantuche (2005).

4. See, e.g., Flaschel (1980, pp. 120-121), Bidard and Erreygers (1998, pp. 434-436) and Lager (2007). It has to be noted, however, that some of the 'joint' products that appear in the SUT may result from statistical classification and, therefore, they do not correspond with the notion of joint production (see, e.g., Semmler, 1984, pp. 168-169; United Nations, 1999, p. 77).

5. A system is said to be 'square' if the number of produced commodities equals the number of operated processes (or industries).

6. For this concept, see Steedman (1975, 1976). It should be noted that the vector of additive labour values (or the labour-commanded prices corresponding to zero profits) does not represent the 'labour costs' of commodities, i.e., the quantities which directly and indirectly have gone to produce them, but rather 'employment multipliers' (à la Kahn, 1931; see Sraffa, 1960, §70, and Steedman, 1975, pp. 118-120). As Sraffa (1960, p. 56) stresses: '[I]n the case of joint-products there is no obvious criterion for apportioning the labour among individual products, and indeed it seems doubtful whether it makes any sense to speak of a *separate* quantity of labour as having gone to produce one of a number of *jointly* produced commodities.' For an attempt to determine labour costs, which is based on the 'Industry Technology Assumption' and the 'Market or

Sales Value Method' (and, therefore, involves a conversion of the original system into a *single* production system), see Flaschel (1980, pp. 121-126; 1983, pp. 443-450).

7. A joint production system is said to be (i) strictly viable if there exists a semi-positive intensity vector such that the net output is positive; and (ii) strictly profitable if there exists a semi-positive price vector such that every industry yields positive profits.

8. See endnote 6.

9. It goes without saying that the SUT are not necessarily square (see, e.g., United Nations, 1999, p. 86, §4.41; Eurostat, 2008, p. 295, §11.1).

10. We hypothesize that wages are paid *ante factum* (for the general case, see Steedman, 1977, pp. 103-105) and that there are no savings out of this income in order to follow most of the empirical studies on this topic.

11. See Filippini and Filippini (1982), Fujimoto and Krause (1988) and Hosoda (1993).

12. See Steedman (1977, chs 12-13; 1992), Kurz and Salvadori (1995, ch. 8) and Bidard (1997).

13. Through to the year 1999, the SUT of the German economy describe 59 products and 60 industries. The German SUT for the years from 2000 onwards are revised and they are not comparable with those of preceding years.

14. For these two different, in the general case, approaches to fixed capital, see Kurz and Salvadori (1995, chs. 7-9) and Bródy (1970, ch. 1.2), respectively.

15. *Mathematica 5.0* is used in the calculations. The analytical results are available on request from the authors.

16. A commodity is said to be 'separately producible' in system $\{\mathbf{B}, \mathbf{A}\}$ if it is possible to produce a net output consisting of a unit of that commodity alone with a non-negative intensity vector. A system of production is called 'all-productive' if all commodities are separately producible in it. Thus, if $\{\mathbf{B}, \mathbf{A}\}$ is 'all-productive', then $[\mathbf{B} - \mathbf{A}]^{-1} \geq \mathbf{0}$. Furthermore, a process (or industry) is 'indispensable' within a system of production if it has to be activated whatever net output is to be produced. An 'all-productive system' whose processes are all indispensable is called 'all-engaging'. Thus, if $\{\mathbf{B}, \mathbf{A}\}$ is 'all-engaging', then $[\mathbf{B} - \mathbf{A}]^{-1} > \mathbf{0}$ (*ibid.*).

17. The additive labour values of the French, German and Greek economies are reported in the Appendix 1, Tables 1.2, 1.4 and 1.6, respectively.

18. It may be noted that, not quite unexpected, the 'rates of surplus values', $(\mathbf{v}^T \mathbf{d})^{-1} - 1$, are greater than the relevant uniform rates of profits.

19. In contrast with the necessary and sufficient condition for $\mathbf{v} \geq \mathbf{0}$, that for $\{r > -1, \mathbf{p} \geq \mathbf{0}\}$ admits no direct economic interpretation (see Fujimoto and Krause, 1988, pp. 193-194). The eigenvalues of systems (6) are reported in the Appendix 2.

20. The '*d* - distance' is used as a measure of deviation. The '*d* - distance', which has been proposed by Steedman and Tomkins (1998, pp. 381-382), constitutes a

numéraire-free measure of deviation and is defined as

$$d \equiv \sqrt{2(1 - \cos \theta)}$$

where θ is the Euclidean angle between the vectors $\mathbf{x}^T \hat{\mathbf{y}}^{-1}$ and \mathbf{e} , $\mathbf{x}^T (\geq \mathbf{0}^T)$ and $\mathbf{y}^T (> \mathbf{0}^T)$ the two vectors under comparison, and $\hat{\mathbf{y}}$ the diagonal matrix formed from the elements of \mathbf{y} .

21. It may be noted that for $s_w = 0.1$ (=0.2), where s_w represents the fraction of wages saved, the deviation of actual prices of production from additive labour values in the German economy for the year 2000 is almost 16.0 per cent (18.1 per cent), whilst the deviation of market prices from actual prices of production is almost 58.0 per cent (58.4 per cent).

22. To our knowledge, there is no relevant empirical study where the market price-labour value deviation is lower than 7 per cent and greater than 37 per cent.

23. See endnote 16.

24. It is important to note that this attribute of the considered systems is independent of the composition and the level of the real wage rate and, therefore, does not rely on our hypothesis that there are no savings out of wages. Furthermore, since the matrices $[\mathbf{B} - \mathbf{A}]^{-1}$ contain negative elements, it follows that the systems under consideration can be 'r-all-engaging' only for some $r > 0$ (*ibid.*).

25. In that case $\lambda^{-1} - 1$ represents the maximum possible rate of growth (and profits), as defined by v. Neumann (1945), \mathbf{y}^T the associated price vector, and \mathbf{x} the associated intensity vector or, alternatively, the intensity vector of Sraffa's (1960, ch. 8) 'Standard system'.

26. The eigenvalues of the pairs $\{\mathbf{B}, \mathbf{A}\}$ of the French, German and Greek economies are reported in Appendix 3, Tables 3.2, 3.4 and 3.6, respectively.

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