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A Dynamic Efficiency-Wage Model with Continuous Effort and Externalities

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ABSTRACT

This paper provides a general equilibrium efficiency-wage model in which employment evolves according to the rules of the Shapiro-Stiglitz's (1984) shirking model. The proposed framework allows us to endogenise in a continuous manner the effort decision undertaken by the individual worker and it may resolve the indeterminacy arising from a model with exogenous (and constant) effort. Moreover, by exploiting an externality argument, we allow the model to capture different local dynamic patterns in which are found convergent fluctuations and persistent cycles. Finally, we show that in our framework unemployment may actually act as a worker discipline device, i.e., equilibria with higher (lower) unemployment rates are also characterised by higher (lower) effort levels.

1. INTRODUCTION

IT IS WELL KNOWN that the Shapiro-Stiglitz (1984) shirking model does not analyse the effort decision undertaken by the individual worker. Given a positive level of effort, the principles of dynamic programming are used to derive the conditions under which the individual employee finds it profitable to behave properly instead of shirking.[2] The results of this theoretical exercise are widely recognised in the labour economics literature: since firms' monitoring is not perfect, a positive involuntary unemployment rate must persist as an equilibrium phenomenon in order to generate a threat that wards off the possible opportunistic behaviour of the workers. The persistence of equilibrium unemployment is ensured in the (neo)classical way, i.e., by offering to employed workers a real wage higher than the opportunity cost of labour.[3]

More recently, this simple partial equilibrium model has been widely exploited also in the real business cycle (RBC) literature, by aiming at exploring the dynamic implications of non-Walrasian approaches to the labour market. Indeed, the 'belief in genuinely involuntary unemployment' underlying the efficiency-wage argument suggested in the shirking model inspired a lot of RBC contributions. Within this literature, it is common practice to assume that employers adjust real wages in order to satisfy an incentive compatibility constraint, according to which workers provide a certain level of effort. However, these dynamic models usually do not analyse in a satisfactory manner the way in which this level of work intensity is determined and the way in which the economy reacts to variable effort. On the one hand, there are dynamic efficiency-wage models in which workers' effort results in being constant across equilibria and even during adjustment processes. Examples of this constant-effort RBC literature are given by Danthine and Donaldson (1990), Gomme (1999), Alexopoulos (2004) and Nakajima (2005). On the other hand, when the work intensity is allowed to vary, the proposed framework distances from the set-up originally proposed by Shapiro and Stiglitz (1984). To our knowledge, the only varying-effort RBC contribution is given by Uhlig and Xu (1996).

This paper follows a different perspective. First, by following the contribution by Kimball (1994), we derive the employment dynamics that arise from a simplified version of the original shirking model. Thereafter, we complete the picture by adding the preferences of an infinitely-lived union to whom is devolved the effort decision.[4] In our dynamic model, work intensity is the only determinant of the real wage. Therefore, our union is called in to determine continuously the 'dividend' of the asset equation that defines the expected discounted value of the utility of the non-shirking employed worker. By following the principles of optimal control, this task is carried out by balancing at the margin the union utility that arises form higher employment — pushed by higher work intensity — with the cost of providing additional effort.

An intriguing feature of our model is that the union may be 'myopic' when it decides effort. Specifically, this hypothesis is pursued by adding an externality factor on the relevant job acquisition rate. This externality factor enters the model through the detection rate and is assumed to depend on the average levels of (un)employment and effort. As will be shown, by assuming that the union may neglect the effects of its choices on the average rate of (un)employment and work intensity, the model becomes able to capture different local dynamic patterns in which are found convergent fluctuations and persistent cycles (orbits). Moreover, we show that under certain conditions, controlling for the effort resolves the indeterminacy from which is affected the standard model with exogenous (and constant) effort. Finally, our model suggests that unemployment may actually act as a worker discipline device, in the sense that equilibria with higher (lower) unemployment rates are also characterised by higher (lower) effort levels.

This paper is arranged as follows. Section 2 derives the employment dynamics underlying a simplified version of the Shapiro-Stiglitz (1984) shirking model with externalities. Section 3 builds a general dynamic equilibrium model with efficiency wages. Section 4 analyses the equilibrium patterns of effort and (un)employment. Finally, section 5 concludes.

2. The employment dynamics of the shirking model

In this section, by exploiting the results in Kimball (1994), we analyse the out-of-equilibrium dynamics of a simplified version of the Shapiro-Stiglitz (1984) shirking model with externalities.

The economy is assumed to consist of a large number of identical firms. Thereafter, we assume that the instantaneous utility of the individual worker is:

$$U_w(t) = w(t) - e(t) \tag{1}$$

where $w(t)$ is the real wage rate and $e(t)$ is the level of effort.

Let q be the detection rate per unit of time worked and b the instantaneous separation rate. Given (1), the asset equation for a shirking worker is the following:

$$\rho_w V_E^S(t) = w(t) + (b+q)\left(V_u(t) - V_E^S(t)\right) \tag{2}$$

where $V_E^S(t)$ is the expected discounted value of utility for an employed shirker, $V_u(t)$ is the expected discounted value of utility for an unemployed worker and ρ_w is the real interest rate used by workers to discount utility flows.

By contrast, the asset equation for a non-shirking worker is given by

$$\rho_w V_E^N(t) = w(t) - e(t) + (b)\left(V_u(t) - V_E^N(t)\right) \tag{3}$$

where $V_E^S(t)$ is the expected discounted value of utility for a non-shirking worker.

The no-shirking condition (NSC) can be derived by imposing $V_E^N(t) = V_E^S(t)$. However, given that firms do not have to pay a real wage rate higher than the payment for which workers behave properly, the NSC holds as equality all the time.[5] Hence:

$$V_E^N(t) = V_E^S(t) \qquad \text{for all } t \tag{4}$$

By subtracting (3) from (2) and using (4) to simplify yields:

$$V_E^N(t) - V_u(t) = \frac{e(t)}{q} \tag{5}$$

Equation (5) states a well-known result in the efficiency wage literature: as long as $q < +\infty$ (imperfect monitoring) workers strictly prefer employment to unemployment. Therefore, employed workers enjoy rents and unemployment is involuntary.

By assuming that unemployment benefits are zero, the asset equation for an unemployed worker is given by

$$\rho_w V_u(t) = m(t)\left(V_E^N(t) - V_u(t)\right) \tag{6}$$

where $m(t)$ is the job acquisition rate.

Notice that when we assume that the unemployment benefits are zero, the level of effort provided by the individual non-shirking worker coincides with her/his reservation wage. Therefore the difference between $w(t)$ and $e(t)$ can be interpreted as an absolute mark-up over the reservation wage.

By subtracting (6) from (3) and using (5) to simplify, $w(t)$ can be written as:

$$w(t) = \frac{e(t)}{q}\left(b + m(t) + \rho_w\right) + e(t) \tag{7}$$

By normalising the size of the labour force to unity, $m(t)$ can be determined from the employment flow identity, i.e.:

$$\dot{L}(t) \equiv m(t)\left(1 - L(t)\right) - bL(t) \tag{8}$$

where $L(t)$ is the level of employment at the individual firm so that 1-$L(t)$ is the corresponding rate of unemployment.

By rearranging (8) we derive:

$$m(t) = \frac{\dot{L}(t) + bL(t)}{1 - L(t)} \tag{9}$$

By substituting (9) in (7) yields:

$$w(t) = \frac{e(t)}{q}\left(b + \frac{\dot{L}(t) + bL(t)}{1 - L(t)} + \rho_w\right) + e(t) \tag{10}$$

As stated by Kimball (1994), (10) is the dynamic version of Shapiro-Stiglitz's (1984) NSC. Specifically, (10) suggests that the efficiency wage that the firm has to pay in order to excite workers' effort is given by the reservation wage augmented by a term that depends on the degree of labour market tightness.

Equation (10) can be used in order to derive the out-of-equilibrium dynamics of employment, i.e.:

$$\dot{L}(t) = \left(1 - L(t)\right)\hat{m}\left(w(t), e(t)\right) - bL(t) \tag{11}$$

where:

$$\hat{m}\left(w(t), e(t)\right) \equiv q\,\frac{w(t) - e(t)}{e(t)} - \rho_w - b \tag{11a}$$

Note that $\hat{m}(\cdot)$ provides an explicit form for the job acquisition rate.[6]

The solution of the dynamic general equilibrium model will be derived by setting $\rho_w = 0$ and assuming that:

$$\frac{w(t) - e(t)}{e(t)} \equiv (e(t))^a \tag{12}$$

where $a \in (0,1)$.

The expression in (12) suggests that the solution of the dynamic general equilibrium model will be derived by assuming that the relative mark-up over the reservation wage is given by a concave constant-elasticity function that has effort as a unique argument. Moreover, given the hypothesis of profit maximisation, (12) implies that the marginal productivity of labour is constant with respect to employment and the real wage rate is unequivocally determined by the work intensity decision. Simple integration suggests that the production function of the representative firm is the following:

$$Y(e(t), L(t)) = \int_0^{+\infty} w(t)dL(t) = L(t) f(e(t)) + \kappa \tag{13}$$

where $f(e(t)) \equiv e(t)(1 + (e(t))^a) = w(t)$ and κ is a constant of integration.

The specifications in (12) and (13) may seem quite *ad hoc*.[7] However, they allow us to capture at least three interesting elements. First, both effort and labour are essential for production. Indeed, $Y(0, L(t)) = Y(e(t), 0) = 0$. Second, being an exponential, the relative mark-up over the reservation wage is always positive so that unemployment is never voluntary. Third, the wage-effort elasticity is higher than unity and increasing in the effort level. Being locally the inverse, this means that the effort-wage elasticity is lower than the level, satisfying the Solow's (1979) condition and decreasing in the effort level.[8] A formal derivation is given in the Appendix.

2.1 The standard model with externalities

As stated in the introduction, a distinguishing feature of our dynamic efficiency-wage model is the presence of externalities. By following Kehoe *et al* (1992), those externalities enter the model by assuming that the employment dynamics is affected by average levels of effort and (un)employment in the overall economy. In order to fulfil this task, we redefine q in the following way:

$$q = q_0 (1 - \bar{L}(t))^{\beta - 1} (\bar{e}(t))^{\alpha - a} \tag{14}$$

where q_0 is the base-line detection probability, $\bar{L}(t)$ and $\bar{e}(t)$ are, respectively, the average economy-wide level of employment and effort while β and α are positive parameters that can take values higher or lower than unity. Given that the economy is assumed to consist of a large number of identical firms, the single employer takes $\bar{L}(t)$ and $\bar{e}(t)$ as given.[9]

On the one hand, the expression in (14) suggests that whenever β is higher (lower) than unity, the detection probability is a decreasing (increasing) function of the average level of employment. This negative (positive) relationship can be rationalised by assuming the operating of a phenomenon of congestion (economies of scale) in the monitoring technology.[10] On the other hand, whenever α is higher (lower) than a the detection probability is an increasing (decreasing) function of the average level of effort. This positive (negative) relationship can be rationalised by assuming that the higher the average effort level, the higher (lower) the incentive for the firm to employ a more efficient monitoring technology.[11]

By substituting (12) and (14) in (11), assuming that the conditions for a symmetric equilibrium are satisfied, i.e., $(1-\overline{L}(t))=(1-L(t))$ and $\overline{e}(t)=e(t)$, and imposing $\rho_w = 0$ leads to:

$$\dot{L}(t)=(1-L(t))^{\beta} q_0 (e(t))^{\alpha} - b \tag{15}$$

Equation (15) states that aggregate employment increases until the share of unemployed workers that find a job equals the share of full employment layoffs.[12] Moreover, note that the base-line detection probability acts as a positive scaling factor in the matching process: in the logic of the shirking model, higher monitoring implies lower equilibrium unemployment. Therefore, higher levels of q_0 increase the share of workers that find (and lose) a job in each period.

As suggested by Kimball (1994), (15) indicates a lagged employment response to labour market conditions. Whenever workers agree to work harder, the firm's willingness to hire increases. In such a situation, the individual employer will be reluctant to hire new employees since the motivation of workers who realise that the labour market is in a period of boom will be weakened, unless they get extra-high wages. Therefore, each individual firm delays new hirings and this moderates the overall hiring rate in the economy.

Consider the case in which effort is constant, i.e., $e(t)=\hat{e}$, for all t. Denote by L^* the stationary solution of (15) and consider the local dynamics in its neighbourhood. The eigenvalue of the linearised equation is given by:

$$\xi \equiv -\frac{\beta b}{1-L^*} < 0 \tag{16}$$

The proof is given in the Appendix.

Given the negative sign of ξ, L^* is locally asymptotically stable. Notice that in the version of the model with constant effort $L(t)$ is not a predetermined variable.[13] Therefore, at the time in which the model is started, a continuum of equilibrium trajectories exists, each corresponding to a continuum of possible starting values for employment in the neighbourhood of L^*. In other words, the equilibrium trajectory is indeterminate.[14]

3. A GENERAL EQUILIBRIUM DYNAMIC MODEL WITH EFFICIENCY WAGES

In this section we build a general equilibrium dynamic model developed in continuous time in which employment evolves according to (15) and the effort is chosen by a representative union whose behaviour is 'myopic'. By following the 'implicit' programming approach, union myopia arises from the fact that the solution of its optimisation problem depends on parameters that, in turn, depend on the solution to the problem itself . Specifically, we assume that the union chooses the effort without considering the effect of its choices on the average levels of effort and (un)employment, i.e., by taking q as given.

In order to simplify notation, from now on we omit the functional dependence of the variables on time. Thereafter, the preferences of the union are described by the following instantaneous utility function:

$$U(L,e) = \ln L - e \tag{17}$$

Equation (17) is slightly different from (1) and it suggests that employment increases union welfare in a logarithmic way, while the cost of exerting effort is linear and normalised to unity for each unit. Specifically, the union realises that exerting effort is costly for workers but they do not dislike being employed.[16] Therefore, knowing that work intensity is the only determinant of the real wage rate, the union is concerned with the employment level with some risk-aversion.[17]

Given (15) and (17) the 'implicit' union problem is:

$$\begin{gathered} \max_{e} \int_0^{+\infty} \exp(-\rho t)(\ln L - e)dt \\ s.to \\ \dot{L} = (1-L)qe^{a} - b \end{gathered} \tag{18}$$

where $q = q_0\left(1-\overline{L}(t)\right)^{\beta-1}\left(\overline{e}(t)\right)^{\alpha-a}$ and ρ is the rate of time preference of the union.

Notice that our union exploits the 'disciplining' effect of unemployment. Specifically, it chooses effort by leaving the firm setting an efficiency wage such that the chosen work intensity will be effectively provided and observing the resulting employment dynamics. This is the mechanism through which is maintained the 'cooperative' equilibrium between the union and the worker . The aim of this particular hypothesis is to analyse in a simple way the behaviour of an economy in which employment evolves according to rules of the Shapiro-Stiglitz (1984) shirking model and effort is a control variable on a continuous scale.

Finally, notice that being a function of the average levels of effort and (un)employment, the detection rate acts like external factors in the union problem. Therefore its solution is not, in general, Pareto efficient.

3.1 The solution of the ‘implicit’ union problem

The current value Hamiltonian of the ‘implicit’ union problem is given by:

$$H = \ln L - e + \Lambda\left(\left(1-L\right)qe^{a} - b\right) \tag{19}$$

where Λ is the shadow value of employment.[19]

The first-order condition for e is:

$$\Lambda\left(1-L\right)qae^{a-1} = 1 \tag{20}$$

Intuitively, (20) states that the union chooses effort by equalising its marginal cost to the marginal net revenue in utility terms arising from an increase in employment.

Consider a situation of symmetric equilibrium, i.e., a situation in which $\left(1-\overline{L}\right) = \left(1-L\right)$ and $\overline{e} = e$. In this case, solving (20) for Λ leads to:

$$\Lambda = \frac{e^{1-\alpha}}{q_0 a\left(1-L\right)^{\beta}} \tag{21}$$

Differentiating with respect to time yields:

$$\dot{\Lambda} = \frac{1-\alpha}{q_0 a\left(1-L\right)^{\beta} e^{\alpha}}\dot{e} + \frac{\beta}{q_0 a\left(1-L\right)^{1+\beta} e^{\alpha-1}}\dot{L} \tag{22}$$

Given (21) and (22), the effort dynamics are described by:

$$\dot{e} = \frac{e}{1-\alpha}\left(\frac{\dot{\Lambda}}{\Lambda} - \frac{\dot{L}}{1-L}\right) \tag{23}$$

Notice that in order to avoid an explosive dynamics for e, we have to rule out the case in which α is equal to unity.

Along the optimal path, Λ has to satisfy the following differential equation:

$$\dot{\Lambda} = \rho\Lambda - \frac{1}{L} + \Lambda qe^{a} \tag{24}$$

In a symmetric equilibrium, given (21), (23) and (24), the work intensity dynamics are described by:

$$\dot{e} = \frac{e}{1-\alpha}\left(\frac{q_0\left(1-\beta\right)e^{\alpha}}{\left(1-L\right)^{1-\beta}} - \frac{q_0 a\left(1-L\right)^{\beta}}{Le^{1-\alpha}} + \frac{\beta b + \rho\left(1-L\right)}{1-L}\right) \tag{25}$$

Notice that (12) allows us to derive an explicit dynamic law also for the real wage rate.[20]

Finally, the transversality condition is given by:

$$\lim_{t\to+\infty} \exp(-\rho t)\Lambda(t)L(t) \tag{26}$$

3.2 Steady-state

In a symmetric equilibrium, i.e., whenever $(1-\bar{L})=(1-L)$ and $\bar{e}=e$, the equality $\dot{L}=0$ implies:

$$e=\left(\frac{b}{q_0(1-L)^{\beta}}\right)^{\frac{1}{\alpha}} \tag{27}$$

In our model, the real wage rate depends only on effort. Therefore, being upward sloped with respect to employment, (27) provides a particular version of the 'canonical' NSC.

The other steady-state condition is derived by imposing $\dot{e}=0$. Hence,

$$\frac{e}{1-\alpha}\left(\frac{q_0(1-\beta)e^{\alpha}}{(1-L)^{1-\beta}}-\frac{q_0 a(1-L)^{\beta}}{Le^{1-\alpha}}+\frac{\beta b+\rho(1-L)}{1-L}\right)=0 \tag{28}$$

There is not an explicit solution for *e* in (28). However, by exploiting (27), it is possible to derive the following expression:

$$G(L)\equiv\frac{b^{\frac{1}{\alpha}}\left(\rho+\frac{b}{1-L}\right)}{q_0^{\frac{1}{\alpha}}(1-\alpha)(1-L)^{\frac{\beta}{\alpha}}}-\frac{ab}{(1-\alpha)L}=0 \tag{29}$$

Consider the case α >(<)1, so that (1-α)<(>)0:

$$\lim_{L\to0^+} G(L)=-(+)\infty \quad \text{and} \quad \lim_{L\to1^-} G(L)=+(-)\infty \tag{30}$$

Since *G*(*L*) is continuous in the interval (0,1) a meaningful stationary solution (*L**, *e*) exists and it is unique. Obviously, *u**= (1-*L*) denotes the equilibrium unemployment rate.

3.3 Local dynamics

Consider the system of non-linear differential equations given by (15) and (25). In a symmetric equilibrium, i.e., whenever $\left(1-\bar{L}\right)=\left(1-L\right)$ and $\bar{e}=e$, its Taylor linear expansion around (*L*, e**) is given by:

$$\begin{pmatrix} \dot{L} \\ \dot{e} \end{pmatrix} = \begin{bmatrix} -\frac{\beta b}{u^*} & \alpha q_0^{\frac{1}{\alpha}} \left(u^*\right)^{\frac{\beta}{\alpha}} b^{\frac{\alpha-1}{\alpha}} \\ j_{2,1} & j_{2,2} \end{bmatrix} \begin{pmatrix} L-L^* \\ e-e^* \end{pmatrix} \tag{31}$$

where:

$$j_{2,1} \equiv \frac{ab\left(1-L^*\left(1-\beta\right)\right)}{u^*\left(1-\alpha\right)\left(L^*\right)^2} + \frac{b^{\frac{1+\alpha}{\alpha}}\left(\left(1-\beta\right)^2+\beta\right)}{\left(1-\alpha\right)q_0^{\frac{1}{\alpha}}\left(u^*\right)^{\frac{2\alpha+\beta}{\alpha}}} \tag{32}$$

$$j_{2,2} \equiv \frac{b\left(1-\beta\right)\left(1+a\right)+\beta b+\rho u^*}{\left(1-\alpha\right)u^*} - \frac{a\alpha q_0^{\frac{1}{\alpha}}\left(u^*\right)^{\frac{\beta}{\alpha}}}{b^{\frac{1-\alpha}{\alpha}}\left(1-\alpha\right)L^*} \tag{33}$$

The derivation of the elements of the Jacobian matrix in (31) is given in the Appendix.

It is well-known that the trace TR(*J*) and the determinant DET(*J*) of the Jacobian matrix arising from a linearised dynamic system represent, respectively, the sum and the product of its eigenvalues. By exploiting (31)-(33), they are given by the following expressions:

$$\mathrm{TR}\left(J\right)=\frac{b\left(1-\beta\right)\left(1+a\right)+\beta b+\rho u^*}{\left(1-\alpha\right)u^*} - \frac{a\alpha q_0^{\frac{1}{\alpha}}\left(u^*\right)^{\frac{\beta}{\alpha}} b^{\frac{\alpha-1}{\alpha}}}{\left(1-\alpha\right)L^*} \tag{34}$$

$$\mathrm{DET}\left(J\right)=-\frac{\beta b}{u^*}\left(\mathrm{TR}\left(J\right)+\frac{\alpha\beta b}{u^*}\right)-\frac{\alpha q_0^{\frac{1}{\alpha}}\left(u^*\right)^{\frac{\beta}{\alpha}}\left(b\left(1-\beta\right)\left(1-a\right)+\beta b+\rho u^*\right)}{b^{\frac{1-\alpha}{\alpha}}\left(1-\alpha\right)u^*} \tag{35}$$

Given (34) and (35), it should be clear that an exact analysis of the sign of TR(*J*) and DET(*J*) done without specifying the way in which each parameter affect the equilibrium rate of unemployment (and the equilibrium level of effort) risks being ineffective. However, it is clear that particular attention should be devoted to the effects of different values of α and β, each of them chosen in the close neighbourhood of unity. Specifically, suppose β is kept fixed and allow α to vary. If the variations of α lead TR(J) to change sign by neutralising systematically the additive effect of the term in bracket on RHS of (35), the 'symmetric' elasticity of the relative mark-up over the reservation wage with respect to effort would distinguish the case in which TR(J) is positive and DET(J) is negative from the case in which the reverse occurs. In the former case, the steady-state is a saddle point, in the latter a sink. In order to verify this and other possibilities, we resort to some numerical examples.

3.4 Numerical simulations

In order to verify the analytical insights put forward above, we perform some numerical simulations of the model, aimed at exploring the dynamic behaviour of the differential equations system given by (15) and (25) for different values of α and β in the neighbourhood of unity.[21] The other parameters are calibrated by following similar contributions.[22] Specifically, we set $q_0 = 1$, $\alpha = 0.6$ and $\rho = 0.03$.

First, we consider the effects of different values of α, i.e., the parameter that summarises the effect of effort on the detection probability and on the matching function. By setting $\beta = 0.8$, we obtain the results in table 1.

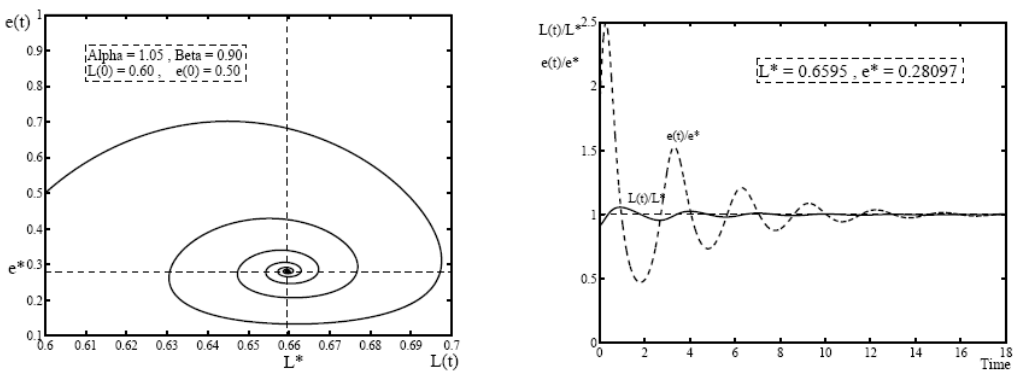
Table 1: Simulation results for different values of α (β=0.8)

α	ξ_1	ξ_2	*Dynamics*
0.8	1.3752	-0.97861	*Saddle*
0.9	1.98	-1.2715	*Saddle*
0.99	9.0597	-2.6576	*Saddle*
1.01	$-3.1302+3.7527i$	$-3.1302-3.7527i$	*Sink*
1.05	$-0.59886+2.0852i$	$-0.59886-2.0852i$	*Sink*
1.1	$-0.28325+1.4956i$	$-0.28325-1.4956i$	*Sink*
1.2	$-0.12707+1.0568i$	$-0.12707-1.0568i$	*Sink*

On the one hand, the results in table 1 suggest that α discriminates between oscillatory (complex) and non-oscillatory dynamics. Specifically, for $\alpha < 1$ the Jacobian matrix in (31) displays two real roots of opposite sign. This means that the steady-state is a saddle point. As suggested by Georges (1995), this kind of local dynamic resolves the indeterminacy of the standard model with constant effort. Indeed, by representing a one-dimensional stable manifold, a saddle path implies that the trajectory converging to the steady-state is unique while all the others diverge. In this case, for every $L(0)$ in the neighbourhood of L^* there will be a unique $e(0)$ in the neighbourhood of e^* that generates a trajectory converging to (L^*, e^*). This value of $e(0)$ should be selected in order to verify the transversality condition in (26) and it will place the system on the stable branch of the saddle path. Therefore, whenever the steady-state is given by a saddle point, the equilibrium path is locally determinate.[23]

On the other hand, whenever $\alpha > 1$ the Jacobian matrix in (31) displays two complex conjugate roots. Therefore, the adjustment of employment and effort occurs thorough oscillations. However, the real part of the complex roots is negative and this means that the steady-state is a sink, so that oscillations are convergent. In a bi-dimensional space, a sink represents the case of indeterminacy. In this case, indeed, the stable manifold has the same dimension of the space of variables. Therefore, there will be a continuum of equilibrium paths $\{L(t),\ e(t)\}$, indexed by $e(0)$, since any path converging to the steady-state (L^*, e^*) necessarily satisfies the transversality condition in (26). In other words, in the neighbourhood of (L^*, e^*) all the trajectories are optimal. See figure 1.

Figure 1: The sink



An indeterminate equilibrium path allows for the possibility of business cycles driven by self-fulfilling beliefs.[24] In this case, indeed, the fundamentals of the economy are not able to pin down a unique equilibrium path. Therefore, whenever the conditions for indeterminacy are satisfied, the description of the environment provided by the union problem above is somehow incomplete. What we could have missed is the forecasting rule used by agents to predict the future. Indeed if, in the neighbourhood of the steady-state all the trajectories are optimal, the path actually followed by the economy could be the one that — consistent with the final equilibrium position (L^*, e^*)— allows for the continuous validation of the agents' prophecies.[25]

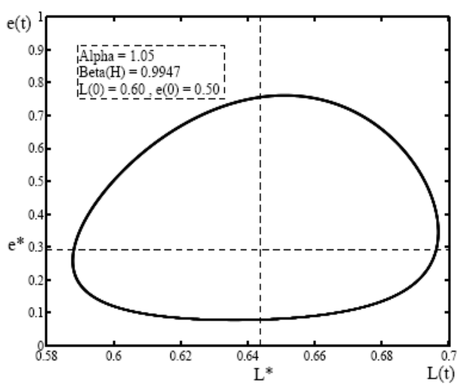
Now we perform some numerical simulations by fixing the value of α but allowing β, i.e., the parameter that summarises the effect of (un)employment on the detection probability and on the matching function, to vary. The most interesting results can be obtained by considering the case of oscillatory dynamics, i.e., the case in which α is higher than unity.[26] Specifically, by setting $\alpha = 1.05$, we obtain the results in table 2.

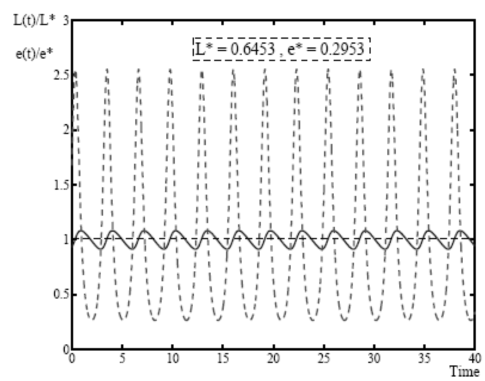
Table 2: Simulation results for different values of β

β	ξ_1	ξ_2	*Dynamics*
0.8	-0.59886+2.052i	-0.59886-2.052i	*Sink*
0.9	-0.2775+2.1158i	-0.2775-2.1158i	*Sink*
0.99	-0.011406+2.1071i	-0.011406-2.1071i	*Sink*
0.9947	+2.1059i	-2.1059i	*Orbit*
1	0.015547+2.1046i	0.015547-2.1046i	*Source*
1.05	0.15386+2.0862i	0.15386-2.0862i	*Source*
1.1	0.2865+2.0605i	0.2865-2.0605i	*Source*
1.2	0.53823+1.9888i	0.53823-1.9888i	*Source*

The results in table 2 suggest that inside the oscillatory dynamics, β is a bifurcation parameter for our dynamic system. Specifically, whenever $\beta <$ 0.9947 the real part of the two complex conjugate roots is negative. As stated above, this implies that the steady-state is a sink. By contrast, whenever $\beta =$ 0.9947 the two complex roots become purely imaginary, suggesting the occurrence of a Hopf bifurcation.[27] In this case the optimal trajectory is given by a closed orbit, i.e., a limit cycle. See figure 2.

Figure 2: The limit cycle





Finally, whenever $\beta > 0.9947$ the real part of the complex roots becomes positive. This means that the steady-state is a source, i.e., locally unstable.[28]

The occurrence of a persistent cycle is an interesting feature of our dynamic efficiency-wage model.[29] Remember that in (12) we assumed that the real wage rate is a non-linear function of effort only. Therefore, whenever the optimal trajectory is given by a limit cycle, a similar relationship holds also between $L(t)$ and $w(t)$. Given that such a circular relationship implies a very low degree of correlation between wage and employment, this kind of dynamic macroeconomic behaviour has been suggested as a rationalisation for the Dunlop-Tarshis observation according to which real wages are acyclical. See, for example, Coimbra (1999).

Before closing this section, we note from the simple observation of figures 1 and 2 that in our model the fluctuations of $e(t)$ — and therefore the fluctuations of $w(t)$ — are wider than the fluctuations of $L(t)$. As suggested by Danthine and Donaldson (1990), this result is at odds with the empirical evidence and it should be due to the particular manner in which effort determines the mark-up over the reservation wage in (12). Indeed, whenever $\alpha > 1$, the production function in (13) exhibits a strong degree of increasing returns and this amplifies the volatility of the real wage.[30] However, the comparison of the different volatility degrees displayed by $w(t)$ and $L(t)$ is beyond the scope of our contribution.

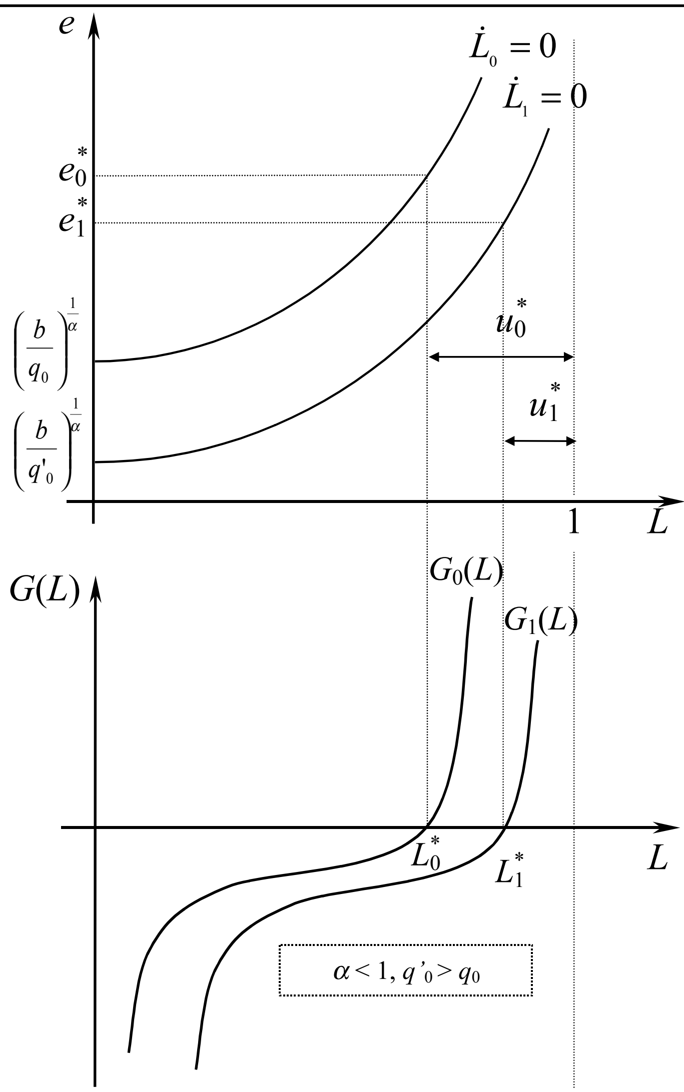
4. Is Equilibrium unemployment a worker discipline device?

Whenever it is possible to find a path converging to the unique steady-state (L^*, e^*), our model can be exploited for comparative statics. Specifically, our framework can be used to analyse the equilibrium effect of an improved monitoring technology, i.e., the effect of an increase in the base-line monitoring probability q_0.

Consider the case in which α is lower than unity.[31] Given (27) and (29), higher values of q_0 lead the locus $L = 0$ to shift downward, while $G(L)$ shifts to the right. Therefore, the new equilibrium will be characterised by higher employment and lower effort. See the two panels of figure 3.

The result illustrated in figure 3 suggests that in our model effort is countercyclical, i.e. equilibria characterised by higher (lower) unemployment rates are also characterised by higher (lower) effort levels. As in the original contribution by Shapiro and Stiglitz (1984), this is consistent with the idea that unemployment acts like a threat and that threat is more pronounced when unemployment is high.[32] This possibility is found also in Uhlig and Xu (1996).

Figure 3: The 'disciplining' effect of unemployment



5. Concluding remarks

This paper provided a dynamic efficiency-wage model that allows us to endogenise in a continuous manner the effort decision undertaken by the individual worker in the Shapiro-Stiglitz (1984) model. Specifically, we derived the employment dynamics that arises from a simplified version of that partial equilibrium model by assuming that the relative mark-up over the reservation wage is given by a concave constant-elasticity function of work intensity.

Thereafter, we completed the picture by adding the preference of a union to whom is devolved the effort decision. Moreover, we allowed this decision process to be blurred by an externality that affects the relevant job acquisition rate through the monitoring technology. This external factor allows for the possibility of 'myopic' union behaviour.

The solution of the resulting dynamic model shows that there may be an interesting interaction between the matching and the monitoring technologies. Specifically, whenever the matching and the monitoring technologies combine in order to deliver a matching function that displays decreasing returns with respect to effort, the steady-state equilibrium is a saddle point. In our model this happens whenever the elasticity of the relative mark-up over the reservation wage with respect to effort is lower than unity. In this case, given the saddle-path dynamics, controlling for work intensity resolves the indeterminacy from which is affected the standard model with constant (and exogenous) effort.

By contrast, we show that whenever the matching and the monitoring technologies deliver a matching function that displays increasing returns with respect to effort, the dynamics of the model become oscillatory: effort (and the real wage together) and employment fluctuate over time. In our framework this happens whenever the elasticity of the relative mark-up over the reservation wage with respect to effort is higher than unity. Within oscillatory dynamics, the way in which the unemployment rate affects the monitoring and matching process has been shown to be essential in order to distinguish between stability and instability. Specifically, we show that whenever the matching and the monitoring technologies deliver a matching function with an elasticity with respect to unemployment lower (higher) than a given threshold quite close to unity, the local dynamic of the model is stable (unstable). As stressed in the literature on self-fulfilling prophecies, the case of complete stability (sink) allows us to explain business fluctuations driven by the self-fulfilling beliefs of agents and may justify the 'animal spirit hypothesis' of business cycles.[33]

Moreover, we show that in our model there is a combination of parameter values such that the optimal trajectory is given by a closed orbit (limit cycle) generated by a Hopf bifurcation. Given that in our model effort is the only determinant of the real wage rate, we show that such a circular relationship may hold also between wages and employment. This kind of local dynamic has been suggested to explain the low degree of correlation between employment and real wages originally observed by Dunlop (1938) and Tarshis (1939).

Finally, we find that under our specification, effort is countercyclical. As in the original contribution by Shapiro and Stiglitz (1984), unemployment acts like a threat and that threat is more effective when the rate of unemployment is high. Therefore, stable equilibria characterised by higher (lower) unemployment rates are also characterised by higher (lower) work intensity.

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APPENDIX

In this appendix some of the equations used in the main text are explicitly derived.

The Wage-Effort Elasticity

Given (12), the wage-effort elasticity is the following:

$$\varepsilon_{w,e} \equiv \frac{\partial w}{\partial e}\frac{e}{w} = 1 + \Omega(e) > 1 \tag{A1}$$

where $\Omega(e) \equiv \dfrac{ae^a}{1+e^a}$

Obviously, whenever $e > 0$, $\Omega'(e) > 0$

Q.E.D.

The Eigenvalue of the Standard Model with Externalities

Whenever $e = \hat{e}$ for all t, the equality $\dot{L}(t) = 0$ implies:

$$\hat{e} = \left(\frac{b}{q_0 \left(1 - L^*\right)^\beta} \right)^{\frac{1}{\alpha}} \tag{B1}$$

where L^* is the stationary solution for employment.

Thereafter:

$$\left.\frac{\partial \dot{L}(t)}{\partial L}\right|_{L=L^*} = -\frac{\beta b}{1-L^*} \equiv \xi \tag{B2}$$

The Jacobian Matrix

In a symmetric equilibrium, i.e., whenever $(1-\bar{L}) = (1-L)$ and $\bar{e} = e$, the elements of the Jacobian matrix in (31) are given by the derivatives with respect to L and e of (15) and (25) evaluated in (L^*, e^*) . The derivatives are the following:

$$\frac{\partial \dot{L}}{\partial L} = -\beta\left(1-L\right)^{\beta-1} q_0 e^{\alpha} \tag{C1}$$

$$\frac{\partial L}{\partial e} = \alpha\left(1-L\right)^{\beta} q_0 e^{\alpha-1} \tag{C2}$$

$$\frac{\partial e}{\partial L}=\frac{q_0 a\left(1-L\left(1-\beta\right)\right)}{\left(1-\alpha\right)L^2\left(1-L\right)^{1-\beta}}+\frac{\left(1-\beta\right)q_0 e^{\alpha+1}}{\left(1-\alpha\right)\left(1-L\right)^{2-\beta}}+\frac{e\beta b}{\left(1-\alpha\right)\left(1-L\right)^2} \tag{C3}$$

$$\frac{\partial \dot{e}}{\partial e}=\frac{\rho}{1-\alpha}-\frac{\left(1-L\right)^{\beta} q_0 a\alpha e^{\alpha-1}}{\left(1-\alpha\right)L}+\frac{q_0\left(1-\beta\right)\left(1+\alpha\right)e^{\alpha}}{\left(1-\alpha\right)\left(1-L\right)^{1-\beta}}+\frac{\beta b}{\left(1-\alpha\right)\left(1-L\right)} \tag{C4}$$

Exploiting (27), it is possible to derive the expressions in the main text, i.e.,

$$\left.\frac{\partial \dot{L}}{\partial L}\right|_{L=L^*,e=e^*}=-\frac{\beta b}{u^*} \tag{C5}$$

$$\left.\frac{\partial \dot{L}}{\partial e}\right|_{L=L^*,e=e^*}=\alpha q_0^{\frac{1}{\alpha}}\left(u^*\right)^{\frac{\beta}{\alpha}} b^{\frac{\alpha-1}{\alpha}} \tag{C6}$$

$$\left.\frac{\partial e}{\partial L}\right|_{L=L^*,e=e^*}=j_{2,1}=\frac{ab\left(1-L^*\left(1-\beta\right)\right)}{u^*\left(1-\alpha\right)\left(L^*\right)^2}+\frac{b^{\frac{1+\alpha}{\alpha}}\left(\left(1-\beta\right)^2+\beta\right)}{\left(1-\alpha\right)q_0^{\frac{1}{\alpha}}\left(u^*\right)^{\frac{2\alpha+\beta}{\alpha}}} \tag{C7}$$

$$\left.\frac{\partial e}{\partial e}\right|_{L=L^*,e=e^*}=j_{2,2}=\frac{b\left(1-\beta\right)\left(1+\alpha\right)+\beta b+\rho u^*}{\left(1-\alpha\right)u^*}-\frac{a\alpha q_0^{\frac{1}{\alpha}}\left(u^*\right)}{\left(1-\alpha\right)L^* b^{\frac{1-\alpha}{\alpha}}} \tag{C7}$$

where $u^* = 1 - L^*$

Q.E.D.

ENDNOTES

1. Research Fellow at the Department of Economics, University of Pisa, via F. Serafini n. 3, 56124 Pisa (Italy), Tel. +39 050 2212434, e-mail guerrazzi@ec.unipi.it This paper draws on Chapter 3 of my doctoral dissertation at the Department of Economics of the University of Pisa. I'm particularly indebted with Fabio Montemurro for his essential advice in the numerical simulation of the dynamic system derived in the model. I'm also grateful to Davide Fiaschi, Fausto Gozzi and Nicola Meccheri for a number of useful discussions. The usual disclaimer applies.

2. Static models of efficiency wages, in which workers' effort is allowed to vary on a continuous scale, are given by Pisauro (1991) and Allgulin and Ellingsen (2002). Moreover, within contract theory, the contribution by MacLeod and Malcomson (1989) provides a

rationalisation for efficiency-wage models in which continuous effort-wage profiles are the result of a self-enforcing implicit contract between the firm and its employees.

3. Following this perspective, the shirking motivation for the payment of efficiency wages provides also a rationalisation for wage stickiness in the presence of involuntary unemployment. See, for example, Mankiw and Romer (1991).

4. The hypothesis that a union is called to choose the work intensity of the labour force recurs in several bargaining contributions. See, for example, Moene (1988), Cramton and Tracy (1992) and Holden (1997).

5. This statement is equivalent to the standard assumption that an indifferent agent takes the action that the principal favours. See Allgulin and Ellingsen (2002).

6. An interesting re-reading of the shirking model in terms of the search-matching framework proposed by Diamond (1982), Mortensen (1982) and Pissarides (1985) is given by Mortensen (1988).

7. If we assume that there is a fixed capital-labour ratio, the production function in (13) is qualitatively similar to the specification used in Gomme (1999).

8. Moreover, with this specification the equilibrium efficiency wage will be higher than the level, such that the effort-wage elasticity is equal to unity. Indeed, Solow's (1979) efficiency-wage model does not contemplate shirking. Therefore, if additional costs related to low-effort are taken into consideration, this will result in an equilibrium effort-wage elasticity lower than unity. This possibility is shown in a dynamic partial equilibrium model proposed by Faria (2000).

9. In other words, they represent external effects that are not traded in the market.

10. This means that the higher (lower) the number of workers, the hardest (easier) the monitoring.

11. Notice that in case of 'average' full employment and/or whenever the average level of effort is zero, the expression in (14) implies that the rent of employed workers tends to infinity. See equation (5).

12. The same qualitative conclusion is stated in Georges (1995, 2002).

13. 'Since $\dot{L}$ enters the structural model as an expectation, L is nonhistorical.' Georges (1995) pg. 43.

14. Notice that the higher ξ, the faster the speed of convergence to the stationary solution. Therefore, the higher the separation rate and/or the lower the equilibrium unemployment rate, the faster the convergence to L^*.

15. See Kehoe *et al.* (1992).

16. The individual worker receives the real wage rate $w(t)$ only when employed. Moreover, notice that the maximum of (17) is zero, i.e., full employment with shirking.

17. Within a static efficiency-wage model, Pisauro (1991) shows that a reasonably high risk-aversion is sufficient to ensure that equilibrium effort is a continuous variable.

18. In other words, the workers will always choose the effort level set by the union.

19. Notice that the condition $a \in (0,1)$ ensures the concavity of the Hamiltonian. This guarantees that the solution of (18) is a maximum.

20. Specifically, $\dot{w} = \left(1 + (1+\alpha)e^{\alpha}\right)\dot{e}$.

21. The MAT LAB 6.5 codes are available from the author upon request.

22. See Georges (2002) and Giammarioli (2003).

23. If we think of the relative mark-up over the reservation wage as an employment adjustment cost, controlling for resolves the indeterminacy as long as this cost is concave.

24. See Benhabib and Farmer (1994).

25. The forecasting rule (or belief function) used by agents to predict the future is the tool that allows us to solve path multiplicity. See Benhabib and Farmer (2000).

26. As long as $\alpha < 1$, the steady-state remains a saddle point no matter the value of β.

27. See Gandolfo (1997).

28. In other words, whenever the monitoring and matching technology are too sensitive to unemployment the model becomes explosive.

29. Using the discount rate as a bifurcation parameter, a similar result is shown in Georges (2002) within a model without microfoundations. By contrast, given particular parameter values, the limit cycle exhibited in our model is the result of optimising behaviour.

30. The same counter-evident result is achieved by Gomme (1999) in his RBC efficiency-wage model with workers' moderate risk aversion.

31. We have the same results even for $\alpha > 1$.

32. As suggested by Bowles (1985), this is distinguishing features also for Marxian models.

33. See Farmer and Guo (1994).

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