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Expanding Product Variety and Human Capital Formation in an Ageing Economy

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ABSTRACT

Population ageing is a significant demographic phenomenon facing many countries. The present paper aims to ascertain the relationship between population ageing and macroeconomic performance, within the framework of an economic growth model endogenously incorporating innovation and human capital. Consequently, the present model implies that the rate of innovation and the ratio of skilled to unskilled workers would decline over the long run as populations aged further, because of increases in life expectancy. The present model also suggests that, in general, the effect of changes in the retirement age on the rate of innovation and the ratio of skilled to unskilled workers is inconclusive.

1. INTROODUCTION

Population ageing is a serious problem faced by many countries around the world. Particularly in East Asian countries such as Japan, South Korea, Singapore, and Thailand; and across Europe, population ageing in recent years has been far more rapid than in other countries and regions. This has caused concerns regarding possible stagnation of future macroeconomic performance. While population ageing has become a significant phenomenon, as described in terms such as knowledge-based economies, the economic activities of industries in developed countries have become more knowledge-intensive. East Asia, and some other developing countries, have thus in recent years been experiencing a rapid transition to a knowledge-based economy. The most important characteristic of a knowledge-based economy is that innovation for various new products and processes are created through active research and development (R&D), which is necessarily accompanied by technical knowledge of production that will accumulate to promote sustainable growth. When attempting to study economic issues in the phase of a knowledge-based economy; therefore, a model that endogenously incorporates R&D and innovation is required, and then an argument developed from the standpoint of the model.

Many countries have focused recently on the roles played by innovation in economic growth as population ageing progresses. Hence, explaining theoretically the long-term effect of such trends on the innovative performance of each country is an important topic. Therefore, this report analyses mainly the effect of population ageing on macro-level innovation from the perspective of economic growth theory.

Some studies have already addressed the issue of ageing within the framework of economic growth theory. De la Croix and Licandro (1999), for instance, examined the effect of life expectancy on economic growth using an overlapping generations model. Their model implies that for a country with a short life expectancy, an extension of life expectancy would contribute to its economic growth. However, for a country with a life expectancy longer than a certain period, an extension of the life expectancy would affect economic growth negatively. Pecchenino and Pollard (1997) and Futagami and Nakajima (2001) demonstrated a decline in economic growth caused by population ageing based on a model of economic growth with learning-by-doing. Zhang, Zhang and Lee (2001) extended the model of Lucas (1988), emphasising the externality of human capital investment, drew the conclusion that extended life expectancy would affect economic growth positively. Futagami, Iwaisako and Nakajima (2002) used a Romer (1990)-type growth model based on R&D and explained that ageing would have a negative effect on economic growth. As these reports illustrate, perspectives on the effect of population ageing on macroeconomic performance vary depending on the model used.

The present paper develops an argument that is generally in line with these aforementioned studies. However, a major difference from these preceding studies is that we analyse the effect of population ageing on the macro economy, taking into account the roles of both innovation and human capital formation in economic growth. That is because those earlier studies only focused on the roles of either innovation or human capital. For example, the model of de la Croix and Licandro (1999) resembles our model in the assumption that a household selects a period of education before entering the labour market. Unlike our model, however, it fails to consider the roles of innovation through R&D activities. The model of de la Croix and Licandro (1999) also fails to investigate, explicitly, the effect of changes in the timing of workers' retirement on the macro economy. The present paper thus complements and extends these preceding studies.

Our argument will be developed by integrating both innovation and human capital, for several reasons. Modern economies increasingly compel workers to be equipped with higher and more diverse technical skills as industrial knowledge intensity increases. In a knowledge-based economy, the formation of human capital through education and training is thus considered to play more important roles than ever before. Such a trend suggests that to understand the economic implications, there is a need to formulate a model that comprehensively combines human capital and innovation as determinants of economic growth, rather than focusing on either one. Based this perspective of the issue, we examine how population ageing affects the determinants of long-term macroeconomic performance, such as innovation and human capital formation. To address this issue, we construct an economic growth model with endogenous innovation and human capital formation. In the model building, as in Romer (1990), Grossman and Helpman (1991, ch. 3), Barro and Sala-i-Martin (2004, ch. 6), and others, we consider product innovation that shows up as an expansion of the varieties of products.

Our analysis has led to two main results. First, the model implied that the rate of innovation and the ratio of skilled labour to unskilled labour would decline as population ageing progressed, reflecting the extended lifetime. Second, the effect of changes in the retirement age on the rate of innovation and the ratio of skilled to unskilled labour was inconclusive, suggesting that an increase in the retirement age might result in both positive and negative effects on innovation.

The rest of this paper is organised as follows. In Section 2 we build a basic model and examine the subjective equilibrium and market equilibrium. In Section 3 we analyse the effects of population ageing on macroeconomic performance in steady-state equilibrium. Finally, in Section 4 we summarise the main results and conclude.

2. THE MODEL

2.1 The basic structure

We begin by describing the basic structure of our model, which is particularly dependent on the model developed by Grossman and Helpman (1991, ch. 5, section 2). By extending their model, we can examine the relationship between population aging and economic growth. This is an important issue that Grossman and Helpman did not examine.

Consider a closed economy that consists of households and firms. We assume that households (representative individuals with identical preferences and learning abilities) of each age group, all having perfect foresight, are distributed continuously and evenly. In the economy, the life span of all households is given an exogenous fixed value, T, and the retirement age is specified exogenously as a fixed value, Z. In addition, the total population is maintained at an exogenous fixed value of N on the assumption that the number of households expiring naturally at any point in time would equal the number of households being created. Therefore, the density of individuals in the age groups between 0 and T would always be N/T.

In the economy, there are two types of households: skilled labour, with abilities acquired through education and training (human capital), and unskilled labour, without such abilities. In the following discussion, the skilled and unskilled labour types are, respectively, assigned subscripts h and l as unique variables corresponding to each type. As well, unskilled labour is set as numeraire, and its wage rate is normalised as one.

The economy also has two production sectors: consumer goods and the R&D sectors. Unskilled labour is employed only in the consumer goods sector. Skilled labour, on the other hand, is employed only in the R&D sector, just as in Romer (1990). We also assume that a wage rate of $w(t)$ is paid to embodied human capital.

2.2 Consumers

We now consider the households who were born at time κ, called generation κ. For the type i=h,l of generation κ, the lifetime utility takes the form

$$U_i(\kappa) = \int_{\kappa}^{\kappa+T} e^{-\rho(t-\kappa)} \log D_i(\kappa,t) \quad dt, \quad i = h,l. \tag{1}$$

In eq. (1) $D_i(\kappa, t)$ denotes the benefits from various differentiated consumer goods, i.e., the instantaneous utility of households belonging to type i of generation κ at time t. The parameter ρ is the subjective discount rate. Here, we assume that a type of good is continuous and that the set of available goods at time t is specified in the interval $[0,n(t)]$. This means that $n(t)$ is interpreted as the number of the varieties of goods (a measure) existing at time t.

At this point, we shall specify the concrete form of the consumption index, $D_i(\kappa, t)$. In this paper, we formulate the following equation:

$$D_i(\kappa,t) \ = \left[\int_0^{n(t)} x_{ij}(\kappa,t)^{\alpha} dj \right]^{\frac{1}{\alpha}}, \tag{2}$$

where α is a parameter that takes a value between (0,1), and $x_{ij}(\kappa,t)$ expresses the consumption of product j of the households belonging to type i of generation κ at time t. In this case, the elasticity substitution between any two products is $1/(1-\alpha)$.

We turn now to a discussion of the budget constraints that households face. Let us assume that households are able to lend and borrow freely at an interest rate of $r(t)$; no assets or debts will remain at death. Here, we express the total expenditure of the households belonging to type i of generation κ as $E_i(\kappa,t)$, and the sum of the discounted present values of the income flow that the household of type i of generation κ acquires by engaging in labour in the R&D sector or consumer goods sector as $W_i(\kappa)$. Then, the intertemporal budget constraint that the household of type i of generation κ faces is expressed as follows:

$$\int_{\kappa}^{\kappa+T} e^{-\int_{\kappa}^{t} r(s)ds} E_i(\kappa,t) \quad dt \leq W_i(\kappa). \tag{3}$$

The optimisation problem of the households belonging to type i of generation

κ can be considered in the following two stages. The first stage is a problem that seeks the quantity of each type of good demanded that will maximise the instantaneous utility under a given set of expenditure and price. To put it another way, households belonging to type i of generation κ will solve the following problem at time t, given $[0,n(t)]\rightarrow R_{+}$ and $E_i(\kappa,t)$, as

$$\max \quad \left[\int_0^{n(t)} x_{ij}(\kappa,t)^{\alpha}\, dj\right]^{\frac{1}{\alpha}},$$

$$\text{s.t.} \quad \int_0^{n(t)} p_j(t)\; x_{ij}(\kappa,t) \quad dj \leq E_i(\kappa,t).$$

If we solve the optimisation problem, then we get

$$x_{ij}(\kappa,t) \qquad = \frac{p_j(t)^{\frac{1}{\alpha-1}} E_i(\kappa,t)}{\int_0^n p_{j'}(t)^{\frac{\alpha}{\alpha-1}} dj'}. \tag{4}$$

Here, we express the number of households belonging to type i of generation κ in subjective equilibrium as $M_i(\kappa,\ t)(i=h,l)$. Consequently, based on our previous assumption that the age groups are distributed evenly at N/T, the relationship $\Sigma_i\ M_i(\kappa,\ t)=N/T$ holds for all generations at any point in time. In addition, if the aggregate demand of generation κ for product j and the total expenditure of the people of generation κ are expressed as $x_j(\kappa,\ t)$ and $E(\kappa,\ t)$ respectively, then the following relationship can be established:

$$x_j(\kappa,t) = M_h(\kappa,t)x_{hj}(\kappa,t) + M_l(\kappa,t)x_{lj}(\kappa,t), \tag{5}$$

$$E(\kappa,t) = M_h(\kappa,t)E_h(\kappa,t) + M_l(\kappa,t)E_l(\kappa,t). \tag{6}$$

Using eqs. (4), (5), and (6), the aggregate demand of generation κ for product j can be expressed as follows:

$$x_j(\kappa,t) \ = \frac{p_j(t)^{\frac{1}{\alpha-1}} E(\kappa,t)}{\int_0^n p_{j'}(t)^{\frac{\alpha}{\alpha-1}} dj'}.$$

Accordingly, if the aggregate demand of all households for product j at time t and the total expenditure of all households are expressed by $x_j(t)$ and $E(t)$ respectively, then the following equation holds.

$$x_j(t) = \frac{p_j(t)^{\frac{1}{\alpha-1}} E(t)}{\int_0^{n(t)} p_{j'}(t)^{\frac{\alpha}{\alpha-1}} dj'}. \tag{7}$$

Therefore, the price elasticity of the demand for product j proves to be $1/(1-\alpha)$.

The problem at the second stage is one that seeks a spending path that maximises the discounted present value of lifetime utility under intertemporal budget constraints. Substituting from eq. (4) into eq. (2), we obtain

$$D_i(\kappa,t) = \frac{E_i(\kappa,t)}{\left[\int_0^{n(t)} p_j(t)^{\frac{\alpha}{\alpha-1}} dj\right]^{\frac{\alpha-1}{\alpha}}}. \tag{8}$$

Here, we define the price index P_D as

$$P_D \equiv \left[\int_0^{n(t)} p_j(t)^{\frac{\alpha}{\alpha-1}} dj\right]^{\frac{\alpha-1}{\alpha}}.$$

Therefore, eq. (8) can be rewritten as

$$D_i(\kappa,t) = \frac{E_i(\kappa,t)}{P_D(t)}.$$

Hence, in the second stage the optimisation problem of type i households can be expressed as follows:

$$\max \quad \int_\kappa^{\kappa+T} e^{-\rho(t-\kappa)} \left[\log E_i(\kappa,t) - \log P_D(t)\right] \quad dt,$$

$$\text{s.t.} \quad \int_\kappa^{\kappa+T} e^{-\int_\kappa^t r(s)ds} E_i(\kappa,t)\, dt \le W_i(\kappa).$$

This implies that type i households will seek a spending path that maximises indirect utility, under the constraint, that is, eq. (3). Consequently, we obtain

$$\frac{\dot{E}_i(\kappa,t)}{E_i(\kappa,t)} = r(t) - \rho.$$

From eq. (9), we find that the growth rate of the total expenditure of households for all types and generations is determined by the gap between interest rate $r(t)$ and subjective discount rate ρ.

2.3 Producers

The market for consumer goods is characterised by monopolistic competition. Therefore, each firm chooses its profit-maximising price of the consumer good at each point in time. In the present model, only unskilled labour is employed as a factor of production for producing all consumer goods. Let us assume that all firms can produce one unit of consumer goods using one unit of unskilled labour. Accordingly, the profit of the firm that produces product *j* (hereinafter, firm *j*) at any point in time is given by

$$\pi_j(t) = \left[p_j(t) - 1 \right] x_j(t). \tag{10}$$

Under the monopolistic competition condition, firm *j* recognises the structure of demand eq. (7) for its own product. Thus, eq. (10) can be rewritten as

$$\pi_j(t) = \left[p_j(t) - 1 \right] \frac{p_j(t)^{\frac{1}{\alpha-1}} E(t)}{\int_0^n p_{j'}(t)^{\frac{\alpha}{\alpha-1}} dj'}. \tag{11}$$

When firm *j* sets its profit-maximising price, from eq.(11), that price will be obtained as follows:

$$p_j(t) = \frac{1}{\alpha} \equiv p. \tag{12}$$

Because $\alpha \in (0,1)$, the price of firm *j*'s product is greater than the marginal cost. Substituting from eq. (12) into eq. (7), the quantity demanded of product is given by

$$x_j(t) = \frac{\alpha E(t)}{n(t)} \equiv x(t). \tag{13}$$

Furthermore, from eqs. (11), (12), and (13), we obtain the following profit of firm *j* in subjective equilibrium.

$$\pi_j(t) = (1-\alpha) \frac{E(t)}{n(t)} \equiv \pi(t). \tag{14}$$

The positive profit will be paid to shareholders as dividends. Our discussion so far indicates that the product's price, production quantity, and operating profit of each firm in a subjective equilibrium will be at the same level for all firms, thereby confirming that a symmetric equilibrium holds in the consumer goods sector.

Next, we examine the R&D activities of firms that create technical knowledge for a new differentiated product. We assume that the producer of each type of consumer good is also the inventor of associated technological inventions and that each firm maintains permanent patent rights to the invention of the production method for a new consumer good that is developed internally. In other words, technical knowledge for new products is created by R&D departments within the consumption goods firms. We also assume a condition under which all firms possess identical technology for creating the blueprints of goods.

New inventions are to be created according to the following equation:

$$\dot{n}(t) = \frac{K(t)}{\delta} H(t), \tag{15}$$

where $\dot{n}$ expresses the number of new technologies created as a result of R&D activities. Also, $K(t)$ represents the existing public knowledge capital (i.e., technical knowledge stock) and $H(t)$ represents the human capital input. At this time, eq. (15) also implies that the human capital input required for creating the invention of $n(t)$ per unit of time is expressed as $\delta \dot{n}(t)/K(t)$. Other things being equal, therefore, the input required for creating the same number of inventions decreases as the technical knowledge stock accumulates in the economy. This should allow the interpretation that such a trend is a reflection of the spillover effect of technical knowledge stock in the R&D sector. In addition, δ refers to the parameter that affects the level of productivity of R&D activities, which can be interpreted as an indicator that reflects such factors as the completeness of research facilities and the government's policy stance toward scientific technology. Assuming, for instance, that other factors are constant, the smaller the value of δ, the higher the productivity $K(t)/\delta$. This higher productivity will create new technologies for more consumer goods.

In the present model, all firms in the R&D sector are small entities. Given the stock of technical knowledge in the economy, they therefore attempt to maximise profits. Here we follow Grossman and Helpman (1991, ch.3) by assuming the stock of technical knowledge has a one-to-one relationship with the variety of consumer goods existing at each point in time. This means that the relationship of $K(t) = n(t)$ holds. Based on that relationship, if the sum of the discounted present values of profits acquired by a firm through the creation of goods is expressed by $V(t)$, then the following relation holds.

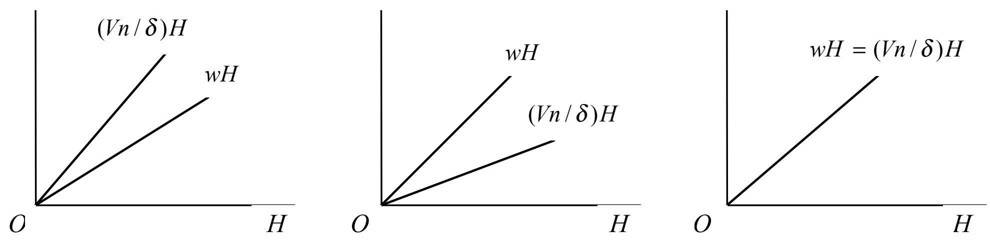
$$V(t) = \int_t^{\infty} e^{-\int_t^{\omega} r(\tau) d\tau} \pi(\omega)$$

Therefore, profit from the R&D activity can be represented as

$$V(t)\dot{n}(t) - w(t)H(t) = \left[V(t) \frac{n(t)}{\delta} - w(t) \right] H(t). \tag{16}$$

We consider that a firm is of arbitrary scale. As shown in Figure 1, the firm's profit is either positive, zero, or negative depending on the relationship between $V(t)n(t)/\delta$ and $w(t)$.

Figure 1: Relationship between revenue and cost by R&D



For example, a firm attempts to invest all possible resources into its R&D activities when $V(t)n(t)/\delta > w(t)$. This means that this case does not hold in subjective equilibrium. On the other hand, if $V(t)n(t)/\delta < w(t)$, then the firm does not invest resources, and consequently it does not create new technologies. In other words, innovation does not occur because $\dot{n}(t) > 0$. Therefore, in the subjective equilibrium of an R&D firm for which free entry to the market is guaranteed and positive innovation in terms of occurs, from eq. (16), we find that the following condition must hold:

$$V(t) = \frac{\delta \quad w(t)}{n(t)}. \tag{17}$$

2.4 Market Equilibrium

We now consider the equilibrium conditions that hold in the asset market, consumer goods market, and the production factor market of unskilled labour and human capital.

First, we examine the equilibrium conditions in the asset market that require the no-arbitrage condition be satisfied. This is explainable as follows: Shareholders of a consumer goods firm receive dividends $\pi(t)$ at time t from the firm's profit. Those shareholders also acquire earnings from capital gains (or incur a capital loss). At this point, we assume that the stock price of the consumer goods firm is equal to the sum of the discounted present values of the profit flow, as in Grossman and Helpman (1991, Ch. 3). Consequently, the stock price accords with $V(t)$ and the total earnings of shareholders at time t become $\pi(t) + \dot{V}(t)$ On the other hand, if funds of $V(t)$ are loaned out, then the earnings will be expressed as $r(t)V(t)$. Hence, in the asset market equilibrium, the following no-arbitrage condition must be satisfied.

$$\pi(t) + \dot{V}(t) = r(t)V(t). \tag{18}$$

Next, we turn to the equilibrium conditions in the consumption goods market. As discussed earlier, in the subjective equilibrium of consumption goods companies, the production quantity of all companies is at the identical level of $x(t)$. Therefore, the aggregate supply of consumption goods at time t proves to be $n(t)x(t)$. As well, from eqs. (7) and (12), the aggregate demand for consumer goods is expressed as $\alpha E(t)$. Thus, the following equation holds for equilibrium in the consumption goods market.

$$\alpha E(t) = n(t)x(t). \tag{19}$$

Finally, we describe the equilibrium conditions in the production factor market. The production factors in the model are unskilled labour and human capital. The market equilibrium conditions of the former and latter are given respectively as follows:

$$n(t)x(t) = L(t), \tag{20}$$

3. Steady state analysis

3.1 Determining the rate of innovation

Let us define the path such that a general equilibrium holds and all economic variables grow at a constant rate as a steady-state equilibrium. We derive and discuss the implications of steady-state equilibrium in the rest of this section.

First, we examine the resource constraints in the economy. Let $\theta(t)$ be the ratio of the sum of those who are attending school along with active and retired skilled labour to the total population. We also define $S(t)$ as the period of time that households allocate to education and training. Hence, the total number of people who are receiving an education or training programme without engaging in labour at time t is expressible as $S(t)\theta(t)N/T$. In addition, the number of people who are working as unskilled labourers in the consumer goods sector without obtaining a school education is indicated as $L(t)$. Therefore, we obtain

$$L(t) = [1 - \theta(t)]\frac{N}{T}Z. \tag{22}$$

Meanwhile, the number of skilled labourers working in the R&D sector after completing their school education is expressed as $[Z - S(t)]\theta(t)N/T$. Based on the above, the total population is expressed by the number of people who are receiving education and training, those who are working in the consumer goods sector, those who are working in the R&D sector, and those who have retired, which are expressed as

$$N = S(t)\theta(t)\frac{N}{T} + L(t) + \left[Z - S(t)\right] \quad \theta(t)\frac{N}{T} + (T - Z)\frac{N}{T}. \tag{23}$$

Next, we assume that the human capital, *h*, of each skilled labourer is a function of the schooling period, *S*. In the present paper, we follow Hall and Jones (1999), Jones (2002), and Papageorgiou and Perez-Sebastian (2006), etc. by formulating the equation of a representative agent's human capital formation as:

$$h = \exp(\eta \cdot S), \tag{24}$$

where η is a positive parameter. From eq. (24), we obtain:

$$\frac{dh}{dS}\frac{S}{h} = \eta S.$$

Hence, we find that parameter η denotes a factor determining the amount of the elasticity of human capital with respect to schooling years, given *S*. Hall and Jones (1999) interpret η in eq. (24) as increments in a worker's efficiency due to an additional year of schooling, which is related to the return to schooling estimated in a Mincerian wage regression. In the present paper, we call the parameter η the efficiency of human capital formation. It follows that aggregate human capital in the economy is given by:

$$H(t) = \left[Z - S(t)\right] \quad \theta(t)\frac{N}{T}e^{\eta \cdot S(t)}. \tag{25}$$

We now examine the relationship among the variables that hold in a steady state equilibrium. At this point, the variables confirmed to take a constant value in the steady-state equilibrium path are expressed without the time symbol *t* to clearly indicate the constant.

From eq. (22), we find that $\theta(t)$ and $L(t)$ become constants, respectively. Therefore, from eq. (23), $S(t)$ also becomes constant. Furthermore, from eqs. (19) and (20), we obtain

$$L = \alpha E(t). \tag{26}$$

Eq. (26) implies that $E(t)$ is constant in a steady-state equilibrium, that is, $E(t) > 0$. In our model, if $r(t) = \rho$ holds, then *E* is in steady-state equilibrium. The proof of this is shown in the Appendix. In the following discussion, we focus on an equilibrium such as $r(t) = \rho$.

Using $r(t) = \rho$ and eq. (18), we obtain

$$\frac{\pi(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)} = \rho. \tag{27}$$

Eq. (27) implies that the profit rate, $\pi(t)/V(t)$, is a constant in a steady-state equilibrium. In this case, if the subjective equilibrium conditions in the consumer goods sector eq. (18) and those in the R&D sector eq. (21) are to be considered, $\pi(t)/V(t)$ can be rewritten as follows:

$$\frac{\pi(t)}{V(t)} = (1-\alpha)\frac{E}{\delta\, w(t)}. \tag{28}$$

Because   is a constant, from eq. (28), we find that $w(t)$, paid to one unit of human capital, also becomes constant. Let $g \equiv \dot{n}(t)/n(t)$ be the rate of innovation in the steady-state equilibrium. Then, taking the relationships among eqs. (17), (26), (27), and (28) into account, we obtain

$$\left(\frac{1-\alpha}{\alpha}\right)\frac{L}{\delta\, w} = g + \rho. \tag{29}$$

We turn now to examine the issue of selection in decision-making concerning the time allocation that each household faces. This selection will determine whether a household receives education and training for a certain period (i.e., investment in human capital), and therefore provides skilled labour in the period thereafter, or whether it provides unskilled labour without making such an investment in human capital.

In the case in which a household engages in unskilled labour, the sum of the discounted present values of the income flow can be expressed as

$$\int_{\kappa}^{\kappa+Z} e^{-\rho(t-\kappa)}\, dt = \frac{1}{\rho}\left(1 - e^{-\rho\cdot Z}\right) \tag{30}$$

If, on the other hand, the household engages as skilled labour, the sum of the discounted present values of the income flow can be expressed as

$$\int_{\kappa+S}^{\kappa+Z} e^{-\rho(t-\kappa)} w e^{\eta\cdot S}\, dt = \frac{1}{\rho}\left(e^{-\rho\cdot S} - e^{-\rho\cdot Z}\right) w e^{\eta\cdot S}. \quad (S \le Z) \tag{31}$$

At this point, those households that come to provide skilled labour under a certain time constraint face the problem of selecting a schooling period that will maximise the sum of the discounted present values of the income flow in eq. (31). Therefore, they must solve the following problem.

$$\max\ \xi(S) = \frac{1}{\rho}\left(e^{-\rho S} - e^{-\rho Z}\right) w e^{\eta\cdot S},$$

$$\text{s.t.} \qquad S \le Z.$$

Here, we consider the feasible set to be $F = \{S \mid 0 < S < Z\}$ so that we can consider an interior solution and express the solution to the above problem as:

$$\hat{S} \in \underset{S \in F}{\arg\max}\, \xi(S).$$

Consequently, we obtain:

$$\hat{S} = \frac{1}{\rho}\left[\log\left(1 - \frac{\rho}{\eta}\right) + \rho Z\right]. \tag{32}$$

From eq. (32), we find that $\hat{S}$ depends on the parameters η, ρ and Z. For simplicity, we define $\varphi(\eta, \rho, Z) \equiv (1/\rho) \cdot [\log(1 - \rho/\eta) + \rho Z]$ in the following discussion.

Each household compares the amount of the sum of discounted present values of the income flow earned as skilled labour and that earned as unskilled labour and then determines which type of labour to provide. This means that households decide whether to provide labour to the consumer goods sector or the R&D sector. Let us recall the assumption that all households have an equal ability to acquire skills. For that reason, if the sums of the discounted present values of the income flow in the two cases were unequal, all households would choose to work in the sector that yields the higher amount. Hence, to ensure that people are engaged in labour in both sectors, the sum of the discounted present values of the income flow earned from working in either sector must equal the other in the equilibrium state. Using eqs. (30) and (31), therefore, the following equilibrium conditions must be satisfied.

$$w = \frac{1 - e^{-\rho Z}}{\left(e^{-\rho \cdot \varphi(\eta, \rho, Z)} - e^{-\rho Z}\right) \cdot e^{\eta \cdot \varphi(\eta, \rho, Z)}}. \tag{33}$$

Consequently, taking eqs. (22), (29), and (33) into consideration, we obtain

$$\left(\frac{1 - \alpha}{\alpha}\right) \frac{(1 - \theta) NZ}{\delta T} \frac{\left(e^{-\rho \cdot \varphi(\eta, \rho, Z)} - e^{-\rho Z}\right) e^{\eta \cdot \varphi(\eta, \rho, Z)}}{1 - e^{-\rho Z}} = \rho + g. \tag{34}$$

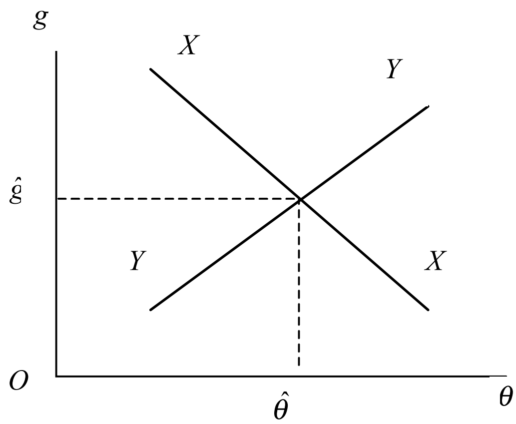
From eqs. (21) and (25), we obtain

$$g = \frac{1}{\delta} \frac{Z - \varphi(\eta, \rho, Z)}{T} \theta N e^{\eta \cdot \varphi(\eta, \rho, Z)}.$$

Therefore, g and θ in a steady state equilibrium will be solved as solutions to eqs. (34) and (35). Figure 2 illustrates the sets of g and θ that satisfy eq. (34)

as line *XX* and the other sets that satisfy eq. (35) as line *YY*.

Figure 2: Steady-state equilibrium of the model



As indicated in Figure 2, the rate of innovation, $\hat{g}$, and the ratio of the sum of those who are attending school along with active and retired skilled labour to the total population, $\hat{\theta}$, in a steady state equilibrium, are determined at the intersection of lines XX and YY. Using eqs.(34) and (35), we obtain the following equations.

$$\hat{g} = \frac{\frac{1-\alpha}{\alpha}\frac{N}{\delta T}\frac{Ze^{\eta\cdot\varphi(\eta,\rho,Z)}[e^{-\rho\cdot\varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{1-e^{-\rho Z}} - \rho}{1+\frac{1-\alpha}{\alpha}\frac{Z[e^{-\rho\cdot\varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{(1-e^{-\rho Z})[Z-\varphi(\eta,\rho,Z)]}}. \tag{36}$$

$$\hat{\theta} = \frac{\frac{1-\alpha}{\alpha}\frac{Ze^{\eta\cdot\varphi(\eta,\rho,Z)}[e^{-\rho\cdot\varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{1-e^{-\rho Z}} - \frac{\rho\delta T}{N}}{[Z-\varphi(\eta,\rho,Z)]e^{\eta\cdot\varphi(\eta,\rho,Z)} + \frac{1-\alpha}{\alpha}\frac{Ze^{\eta\cdot\varphi(\eta,\rho,Z)}[e^{-\rho\cdot\varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{1-e^{-\rho Z}}}. \tag{37}$$

Note that the ratio of skilled labour to unskilled labour *H/L* changes along with the movement of $\hat{\theta}$ in the same direction. Thus, the direction of any changes in the ratio of skilled labour to unskilled labour can be understood by examining the change in $\hat{\theta}$.

3.2 Effects of the process of ageing

In general, population ageing refers to the state in which the percentage of the population of elderly persons (normally 65 years old and above) increases in the total population of a country. In the presented model, if people of age *Z* and above are considered elderly persons, then the ratio of ageing will be provided by 1-Z/T.

This means that the rate of ageing specified by the model depends on the span of life T and retirement age Z. Other things being equal, for instance, the rate of ageing increases when T rises (life expectancy of a given country is extended). On the other hand, an increase in Z (higher retirement age) results in a decreased rate of ageing, holding constant other factors. We will therefore examine how the rate of innovation, $\hat{g}$, and the sum of students and active and retired skilled workers in the total population, $\hat{\theta}$, change in response to changes in lifespan T and retirement age Z, respectively.

Given that the retirement age Z is constant, we analyse the case in which only lifespan T changes. Then, from eqs. (36) and (37), we obtain the following:

$$\frac{\partial \hat{g}}{\partial T} = -\frac{\frac{1-\alpha}{\alpha}\frac{N}{\delta T^2}\frac{Ze^{\eta\cdot\varphi(\eta,\rho,Z)}[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]}{1-e^{-\rho Z}}-\rho}{1+\frac{1-\alpha}{\alpha}\frac{Z[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]}{(1-e^{-\rho Z})[Z-\varphi(\eta,\rho,Z)]}} < 0. \tag{38}$$

$$\frac{\partial \hat{\theta}}{\partial T} = -\frac{\frac{\rho\delta}{N}}{[Z-\varphi(\eta,\rho,Z)]e^{\eta\cdot\varphi(\eta,\rho,Z)}+\frac{1-\alpha}{\alpha}\frac{Ze^{\eta\cdot\varphi(\eta,\rho,Z)}[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]}{1-e^{-\rho Z}}} < 0. \tag{39}$$

Eq. (38) indicates that the rate of innovation is decreased by population ageing, which reflects the lifespan extension. Eq. (39) shows that population ageing reflecting the extended lifespan results in a lower ratio of the sum of those who are attending school along with active and retired skilled labour to the total population, hence a lower ratio of skilled labour to unskilled labour.

$$\begin{aligned}\frac{\partial \hat{g}}{\partial Z} &= \frac{1-\alpha}{\alpha}\frac{N}{\delta T}\frac{e^{\eta\cdot\varphi(\eta,\rho,Z)}[1+z(\eta-\rho)]\cdot[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]}{(1-e^{-\rho Z})^2} \\ &\times\frac{1}{1+\frac{1-\alpha}{\alpha}\frac{Z[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]}{(1-e^{-\rho Z})[Z-\varphi(\eta,\rho,Z)]}} \\ &-\left\{\frac{1-\alpha}{\alpha}\frac{N}{\delta T}\frac{Ze^{\eta\cdot\varphi(\eta,\rho,Z)}[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]}{1-e^{-\rho Z}}-\rho\right\} \\ &\times\frac{1}{\left\{1+\frac{1-\alpha}{\alpha}\frac{Z[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]}{(1-e^{-\rho Z})[Z-\varphi(\eta,\rho,Z)]}\right\}^2} \\ &\times\frac{1-\alpha}{\alpha}\frac{(1-e^{-\rho Z}-\rho Z)[e^{-\rho\cdot\varphi(\eta,\rho,Z)}-e^{-\rho Z}]\cdot[Z-\varphi(\eta,\rho,Z)]}{\{(1-e^{-\rho Z})[Z-\varphi(\eta,\rho,Z)]\}^2}.\end{aligned} \tag{40}$$

$$
\begin{aligned}
\frac{\partial \hat{\theta}}{\partial Z} = & \frac{1-\alpha}{\alpha} \frac{e^{\eta \cdot \varphi(\eta,\rho,Z)}[1+Z(\eta-\rho)] \cdot [e^{-\rho \cdot \varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{(1-e^{-\rho Z})^2} \\
& \times \frac{1}{[Z-\varphi(\eta,\rho,Z)]e^{\eta \cdot \varphi(\eta,\rho,Z)} + \frac{1-\alpha}{\alpha} \frac{Z e^{\eta \cdot \varphi(\eta,\rho,Z)}[e^{-\rho \cdot \varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{1-e^{-\rho Z}}} \\
& - \left\{ \frac{1-\alpha}{\alpha} \frac{Z e^{\eta \cdot \varphi(\eta,\rho,Z)}[e^{-\rho \cdot \varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{1-e^{-\rho Z}} - \frac{\rho \delta T}{N} \right\} \\
& \times \frac{1}{\left\{ [Z-\varphi(\eta,\rho,Z)]e^{\eta \cdot \varphi(\eta,\rho,Z)} + \frac{1-\alpha}{\alpha} \frac{Z e^{\eta \cdot \varphi(\eta,\rho,Z)}[e^{-\rho \cdot \varphi(\eta,\rho,Z)} - e^{-\rho Z}]}{1-e^{-\rho Z}} \right\}^2} \\
& \times \left\{ [Z-\varphi(\eta,\rho,Z)]\eta e^{\eta \cdot \varphi(\eta,\rho,Z)} + \frac{1-\alpha}{\alpha} \frac{(1-e^{-\rho Z} - \rho Z)[e^{-\rho \cdot \varphi(\eta,\rho,Z)} - e^{-\rho Z}] \cdot [Z-\varphi(\eta,\rho,Z)]}{\{(1-e^{-\rho Z})[Z-\varphi(\eta,\rho,Z)]\}^2} \right\}.
\end{aligned}
\tag{41}
$$

Next, we investigate the case in which the retirement age Z increases as the lifespan T remains constant. In this case, from eqs. (36) and (37), we obtain respectively eqs. (40) and (41) above.

As indicated by eqs. (40) and (41), the changes that are imparted on the rate of innovation and the ratio of skilled to unskilled workers by an increase in the retirement age Z are unclear. This suggests that a higher retirement age Z, might cause the rate of innovation and the ratio of skilled to unskilled workers to increase in some cases and decrease in others depending on the relative sizes of the various parameters.

Here, we are concerned in particular with the effect of changes in the retirement age on the rate of innovation. Such an effect might be interpreted intuitively; first, a higher retirement age increases the number of active skilled workers engaging in R&D activities. This has a direct positive effect on innovation. Second, in this model, an increase in the retirement age also increases the number of active unskilled workers. As a result, profit from the production of goods increases, which generates increased R&D revenue to promote the creation of innovation through more active R&D. Third, raising the retirement age means a longer period of time during which skilled workers are able to earn income. Therefore, the incentives for economic agents to undergo additional schooling or training (i.e. invest in their human capital) will increase. The result brings a greater accumulation of human capital in the economy that will increase the rate of innovation.

In addition to these three effects, we note the fact that an extended retirement age affects the relative wages of skilled and unskilled workers. In our model, the effect of an increased retirement age on relative wages is ambiguous: that is, raising the retirement age might increase or decrease the relativities between wages of skilled and unskilled workers, which in this model equal the wage of a skilled worker. Under conditions of increased relative wages along with raising the retirement age, the rate of innovation will also increase because such a change in relativities provides greater incentives to invest in human capital. Thus, when the sign of the effect on relative wages is positive, an increase in the retirement age would cause the rate of innovation to increase, in addition to the three positive effects described above. On the other hand, if the sign of the effect on the relative wages is negative and its absolute value is greater than the sum of the three positive effects, as stated above, then a higher retirement age would cause a decline in the rate of innovation. Specifically, from eq. (40), we find that if $\eta > \rho$, then $\partial \hat{g} / \partial Z > 0$. That is, when the efficiency of human capital formation is greater than the subjective discount rate, an increase in the retirement age promotes the rate of innovation in a steady-state equilibrium.

4. Conclusions

Population ageing is a phenomenon seen in both developed and developing economies. It is an issue that society cannot ignore. Also, in recent years, the importance of human capital and innovation has been recognised increasingly in considerations of long-term macroeconomic growth. Consequently, how population ageing affects the determinants of long-term macroeconomic performance is an extremely important research theme. In the previous section, an economic growth model endogenously incorporating innovation and human capital was built to analyse such issues theoretically.

The following summarises the key analytical results: First, our model implies that the rate of innovation and the ratio of skilled to unskilled workers will decline over the long run with population ageing because of an increase in life expectancy. Second, the effect of changes in the retirement age on the rate of innovation and the ratio of skilled to unskilled workers prove inconclusive. An increase in the retirement age might therefore result in both positive and negative effects on innovation. For example, if the efficiency of human capital formation is greater than the subjective discount rate in a country, then our model suggests that the direction of the effect of an increase in the retirement age on innovative performance is positive. In addition, raising the rate of innovation will make for a higher level of per capita income in the future.

Given the model we present here, it is also worth noting the policy issue of raising the retirement age as it concerns EU countries. In recent years, EU directives have forced countries to address the policy issue of raising the compulsory retirement age, especially given the pressure on pensions across Europe and given other concerns regarding discrimination against older workers. For example, the retirement age for women in the United Kingdom is to be changed from 60 to 65 to bring it into line with that of men. As stated above, our model reflects that when the efficiency of human capital formation is greater than the subjective discount rate in a country, raising the retirement age will affect an increase in the rate of innovation in the macro economy. In our discussion, we considered a steady-state equilibrium such that the subjective discount rate corresponded with the interest rate.

As in Hall and Jones (1999), we can also regard the efficiency of human capital as the rate of return to education. Accordingly, in the context of our model, if the rate of return to education is greater than the interest rate, then implementing a policy of raising the compulsory retirement age will provide an effective contribution to long-term economic performance. For example, Harmon et al (2003) reported the estimates of the rate of return to a year of schooling in the United Kingdom of between 7 and 9 percent for men and between 9 and 11 percent for women. Moreover, the average real interest rate in the UK from 1990 to 2003 was around 3.5 percent (data from the World Bank's World Development Indicators). Based on these figures, the policy of raising the retirement age in the United Kingdom should have a positive effect on macroeconomic performance in the long run.

The present paper examines innovation explicitly through industrial R&D and workers' retirement age, and derives a different implication from that of de la Croix and Licandro (1999) that life expectancy and the economic growth rate have an inverse U-shaped relationship. By comparison, the results of our model resemble the conclusions from studies such as Futagami and Nakajima (2001) and Futagami, Iwaisako and Nakajima (2002), hence supporting their arguments.

As remarked earlier, recent global economic trends suggest the need to develop a framework that is inclusive of both human capital and innovation as determinants of economic growth, rather than concentrating on just one of these, and to draw on the implications of such a model. However, in the context of the economic growth model endogenously incorporating innovation and human capital, few attempts have been made to analyse theoretically the macroeconomic effects of population ageing, taking into account the retirement age of workers. This study, therefore, is considered be a meaningful contribution to research in this area as it presents a model that takes into account the issues of population ageing and innovation.

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APPENDIX

We prove that if $r(t)=\rho$, then E > 0 is in steady-state equilibrium.[6]

In the present model, the aggregate expenditure of all generations at time t is written as:

$$E(t)=\int_{t-T}^{t}E(\kappa,t)d\kappa. \tag{A1}$$

Differentiating eq. (A1) with respect to t and using eq.(9) yields

$$\dot{E}(t) = E(t,t) - E(t-T) + [r(t) - \rho]E(t), \tag{A2}$$

where $E\,(S,t)$ denotes the aggregate expenditure of generation s at time t. From eq. (29), we find that $\dot{E}(t) = 0$ in the steady state equilibrium. Therefore, eq. (A2) can be rewritten as follows:

$$0 = E(t,t) - E(t-T) + [r(t) - \rho]E(t). \tag{A3}$$

Also, if we solve eq. (9), then we obtain

$$E_i(\kappa,t) = E_i(\kappa,\kappa)\cdot e^{\int_\kappa^t [r(t)-s]ds}. \tag{A4}$$

We now focus on the generation of lifetime income for type i of generation k at time t, that is, Wi(k). In equilibrium, the lifetime income of skilled and unskilled agents are equal, and thus $W_l(\kappa) = W_h(\kappa) \equiv \overline{W}(\kappa)$. Hence, the lifecycle budget constraint of generation k is expressed as

$$\int_\kappa^{\kappa+T} E_i(\kappa,t)e^{-\int_\kappa^t r(s)ds}dt = \overline{W}(\kappa). \tag{A5}$$

Substituting eq. (A3) into eq. (A5) yields

$$E_i(\kappa,\kappa) = \frac{\rho}{1-e^{-\rho t}}\overline{W}(\kappa). \tag{A6}$$

Moreover, from eqs. (A4) and (A5), we obtain

$$E_i(\kappa,t) = \frac{\rho}{1-e^{-\rho T}}\overline{W}(\kappa)e^{\int_\kappa^t [r(s)-\rho]ds}. \tag{A7}$$

Therefore, the aggregate expenditure for generation κ, that is, $E(\kappa,\ t)$ is given by

$$\begin{aligned} E(\kappa,t) &= [M_h(\kappa,t)E_h(\kappa,t) + M_l(\kappa,t)E_l(\kappa,t)]e^{\int_\kappa^t [r(s)-\rho]ds}, \\ &= \frac{N}{T}\frac{\rho}{1-e^{-\rho T}}\overline{W}(\kappa)e^{\int_\kappa^t [r(s)-\rho]ds}. \end{aligned} \tag{A8}$$

Using eq. (A8), we can obtain the aggregate expenditure of generation t at time t and one of generation t-T at time t as follows:

$$E(t,t) = \frac{N}{T}\frac{\rho}{1-e^{-\rho T}}\overline{W}(t), \tag{A9}$$

$$E(t-T,t) = \frac{N}{T}\frac{\rho}{1-e^{-\rho T}}\overline{W}(t-T)e^{\int_{t-T}^t [r(s)-\rho]ds}. \tag{A10}$$

Here, if we assume that $r(t) = \rho$, then the lifetime income of an unskilled agent of generation κ; that is, $W_l(\kappa)$, is equal to $(1-e^{-\rho Z})/\rho$, which is equal to $\overline{W}(\kappa)$in steady-state equilibrium. This result implies that

$$\overline{W}(t) = \overline{W}(t-T) = \frac{1-e^{-\rho Z}}{\rho}. \tag{A11}$$

Therefore, when $r(t) = \rho$, from eqs. (A9), (A10) and (A11), we obtain

$$E(t,t) = E(t-T,t) = \frac{N}{T}\frac{1-e^{-\rho Z}}{1-e^{-\rho T}}.$$

Thus eq. (A3) holds. Hence, we find that if $r(t) = \rho$, then E > 0 is in steady-state equilibrium.

ENDNOTES

1. Faculty of Literature and Social Sciences, Yamagata University, 1-4-12 Kojirakawa-machi, Yamagata, Japan. E-mail: noda@human.kj.yamagata-u.ac.jp. I am grateful for the valuable comments of an anonymous referee on an earlier version of the paper. The usual disclaimer applies.

2. Particularly, the so-called work-site abilities, such as skills in information and communication technology, communications skills for collaborative activities, etc., are increasing in importance. See Organisation for Economic Co-Operation and Development (2001) for more details.

3. Jones (2002) analysed the factors of success of the US economy after WWII (1950-93) and found that 80 per cent of that success could be explained by the contributions of R&D and human capital. Such experiences in the growth of the U.S. economy are highly suggestive to many countries that have tried to match the development of the US.

4. The same type of assumption is made in the case of Barro and Sala-i-Martin (2004, Ch. 6), in which various intermediate goods exist.

5. Yusuf and Evenett (2002) emphasise the role of innovation as the key to the sustainable growth of East Asian countries.

6. The proof in the Appendix was suggested by a referee.

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Editor's note: where publishers have preferred the spelling 'aging' this has been left unaltered.

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