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A problem with the course presentation of the single-price alternative to 3rd-degree price discrimination: A generalised version

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1. Initial conditions and standard practice

In standard practice, to establish a basis for comparison, consider conditions in which the demands in markets, R and S, are given, for example, by general, parameterised equations, such as:

$$p_r = a_r - b_r * q_r \tag{1}$$

and

$$p_s = a_s - b_s * q_s \, , \tag{2}$$

where p_r and p_s are prices per unit and q_r and q_s are individual market quantities in markets, R and S, respectively. For ease of exposition, let average and marginal cost be equal to a constant, K. As such, associated profit under 3rd-degree price discrimination, $\pi_{discrimination}$, is defined by the equation:

$$\pi_{discrimination} = p_r * q_r + p_s * q_s - K * (q_r + q_s). \tag{3}$$

Note the various solutions, described below.

For profit-maximisation under 3rd-degree price discrimination, the partial derivatives of the defined profit function with substitution:

$$\pi_{discrimination} = (a_r - b_r * q_r) * q_r + (a_s - b_s * q_s) * q_s - K * (q_r + q_s) \tag{4}$$

with respect to q_r and $q_{s,}$ are set equal to zero and are as given below:

$$\partial \pi_{discrimination} / \partial q_r = a_r - q_r - K = 0 \tag{5}$$

and

$$\partial \pi_{discrimination} / \partial q_s = a_s - q_s - K = 0. \tag{6}$$

Solving:

$$q_r = a_r - K \tag{7}$$

and

$$q_s = a_s - K, \tag{8}$$

provided that q_r and q_s are equal to or greater than 0.

2. A PERFUNCTORY DESCRIPTION OF THE TYPICAL SINGLE-PRICE SOLUTION

Where associated profit is represented by the symbol, $\pi_{typical\ single}$, profit is given by the expression:

$$\pi_{typical\ single} = (a_r - b_r * q_r) * q_r + (a_s - b_s * q_s) * q_s - K * (q_r + q_s). \tag{9}$$

The partial derivatives of the Lagrange-expressed profit function:

$$\pi_{typical\ single} = (a_r - b_r * q_r) * q_r + (a_s - b_s * q_s) * q_s - K * (q_r + q_s) - \lambda * ((a_r - b_r * q_r) - (a_s - b_s * q_s)) \tag{10}$$

with respect to q_r, q_s, and λ, the Lagrange multiplier, set equal to 0 are as given below:

$$\partial \pi_{typical\ single} / \partial q_r = a_r - 2 b_r * q_r - K + \lambda * b_r = 0, \tag{11}$$

$$\partial \pi_{typical\ single} / \partial q_s = a_s - 2 b_s * q_s - K - \lambda * b_s = 0, \tag{12}$$

and

$$\partial \pi_{typical\ single} / \partial \lambda = a_r - b_r * q_r - a_s + b_s * q_s = 0. \tag{13}$$

The Lagrange-expressed profit function, eq. (10), sets $\pi_{typical\ single}$ equal to the sum of revenues less total cost, as defined in eq. (9), and embodies the constraint that the prices in each market must be equal, that is , that $(a_r - b_r * q_r) - (a_s - b_s * q_s) = 0$. Solving:

$$q_r = (a_r - a_s + (b_s/b_r) * ((a_r - K)/2) + (a_s - K)/2)/(b_r + b_s) \tag{14}$$

and

$$q_s = (-a_r + a_s + (b_r/b_s) * ((a_s - K)/2) + (a_r - K)/2)/(b_r + b_s) \quad (15)$$

provided that q_r and q_s are equal to or greater than 0.

3. A more thorough single-price method of solution as an alternative to 3rd-degree price discrimination

The derivative, and this is the important insight in this paper, set equal to 0, as a first consideration, of the one-market profit function,

$$\pi_{alternative\ single} = (a_r - b_r * q_r) * q_r - K * q_r \ , \quad (16)$$

with respect to q_r, only, is as given below,

$$\partial \pi_{alternative\ single} / \partial qr = a_r - q_r - K = 0. \quad (17)$$

Solving:

$$q_r = a_r - K \ , \quad (18)$$

provided that q_r is equal to or greater than 0.

Note, therefore, that by comparison the typical single-price solution can be shown to be unreliable as a measure of the level of maximum profit obtainable for a single-price alternative, in that respective values for the sum of q_r and q_s from the typical solution and for q_r, only, from the substitute solution, result in profits that are greater, for some values of K, but less for others, than that associated with the substitute single-price solution; dependent, of course, on the specific choice of values for the parameters of the cost function, i.e., for values of K, given demands for markets, R and S. As such, for some values of K, the typical single-price solution results in a profit value that is greater than that associated with the substitute single-price solution. For other values, profit for the typical single-price solution is less than that associated with the substitute single-price solution, where q_r, only, is produced. As such, the typical single-price solution and the substitute single-price solution are alternatively unreliable with respect to the level of maximum profit, dependent on the arbitrary, but specific choice of values for K for the cost function, given demands for markets, R and S, and, therefore, at least non-general. More generally, and as instruction for course presentation, where $\pi_{typical\ single}$ is greater than $\pi_{alternative\ single}$, take the typical single-price solution as the solution for the more thorough single-price solution. Where $\pi_{typical\ single}$ is less than $\pi_{alternative\ single}$, replace the typical single-price solution as the solution with the substitute single-price solution for the more thorough single-price solution.

4. The critical value

At this point, profit for the typical single-price solution is now computable by substitution of equations (14):

$$q_r = (a_r - a_s + (b_s/b_r) * ((a_r - K)/2) + (a_s - K)/2)/(b_r + b_s), \quad (14)$$

and (15):

$$q_s = (-a_r + a_s + (b_r/b_s) * ((a_s - K)/2) + (a_r - K)/2)/(b_r + b_s), \quad (15)$$

into equation (9):

$$\pi_{typical\ single} = (a_r - b_r * q_r) * q_r + (a_s - b_s * q_s) * q_s - K * (q_r + q_s), \quad (9)$$

which was given by:

$$\pi_{typical\ single} = K^2 - 20 * K + 100 = 0, \quad (19)$$

in the original article. Profit for the alternative single-price solution is given by equation (16):

$$\pi_{alternative\ single} = (a_r - b_r * q_r) * q_r - K * q_r , \quad (16)$$

which was given by

$$\pi_{alternative\ single} = 0.5 * K^2 - a_r * K + 72 \quad (20)$$

in the original article. Profits, respectively, are equal depending on K.

With respect to the results of the single-price alternative to 3rd-degree price discrimination, profit for the typical single-price solution is greater than profit for the substitute single-price solution for values of average and marginal cost set near, but less than the critical value that equates profits and less than profit for the substitute single-price solution for values of average and marginal cost set near, but greater than the critical value. Profits are equal, of course, for average and marginal cost set equal to the critical value.