

Process Recurrence and Input Use at the Industry Level: A Coherent Long-Period Analysis

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ABSTRACT

The familiar partial equilibrium analysis of an individual industry's use of a particular input involves changing only one price (that of the particular input in question), even when long period equilibrium is considered. But this is incoherent, other than in fluke cases, since such a price change will always force other industries out of long period equilibrium. When this incoherence is removed, and equilibrium is taken seriously, the comparative statics results obtained can differ sharply from those derived from the familiar analysis.

1. INTRODUCTION

Section 2 of this paper will present a number of examples of competitive, constant returns to scale economies in which input use, per unit of output, is positively related to the price of the input. In each case the input use in question will be the direct use of the input in a single-product industry; no reference will be made to any vertically-integrated quantities or to any aggregated, economy-wide quantities. The focus of attention will thus be an individual industry and its use of particular, physically specified inputs. Since some of the examples given may remind the reader of those employed in the discussion of reswitching and capital-reversing, it will be as well not only to stress that our examples focus on the individual industry but also to point out that no value-aggregates of inputs will be considered, that no reswitching of techniques at the economy level will be involved and that, in most of the examples, the rate of interest will be constant (or even constant and zero). While some of the examples will involve industries facing only a limited set of alternative production processes, others will allow for infinitely many such processes in one or more industries. Section 3 of the paper will provide some discussion of the relation between the results of the examples and the familiar analysis of the 'downward sloping input demand curve'; this will lead into a criticism of the way in which the *ceteris paribus* clause is interpreted in that familiar analysis. In both Section 2 and Section 3, all our arguments will be of a strictly comparative statics nature and no process of change will ever be considered.

2. INDUSTRY-LEVEL INPUT USE IN CONSTANT RETURNS ECONOMIES

The reswitching of techniques phenomenon, which we shall not be concerned with in this paper, involves the 'abandonment' and then the 'readoption' of a *complete* set of production process-

es as the rate of interest notionally 'rises'. Suppose that a certain set of processes would be adopted in a competitive economy at an interest rate of two per cent; that many of those processes would not be used at an interest rate of six per cent; and that at an interest rate of 10 per cent *all but one* of the industries would have reverted to the process used at an interest rate of two per cent: this would not be an example of the reswitching of techniques, even if the one industry which had not reverted to its 'original' process were to be just one industry out of a thousand. At the industry level, nevertheless, something noteworthy has occurred in the other 999 industries; in each of them a particular method of production has been 'abandoned' and then 'readopted' as the rate of interest has 'risen' monotonically. We may refer to such an occurrence in an industry as *recurrence* of the production process in that industry. (In this terminology, then, reswitching of techniques requires that at least one industry exhibits recurrence and that every industry exhibits either recurrence or no change of production process).

Of course, in relation to an industry's demand curve for an input, one might be interested not only in the phenomenon of complete (industry) process recurrence but also in the possibility that the amount of some particular input, per unit of output, might 'recur' even while the amounts of other inputs employed do not do so. More generally, one might be interested in checking carefully to see whether the use of particular inputs, per unit of output, is always (weakly) inversely related to the corresponding prices.

The following examples all exclude any reswitching of techniques as the rate of interest varies. While they are in the same spirit as those of Steedman (1985), (1988) and (1998), they give greater emphasis both to the case of a constant - or even zero - rate of interest and to industries with infinitely many alternative methods of production; moreover, models of the Samuelson-Hicks-Spaventa type are here considered for the first time.

Example A

We consider first a simple 'input output' system involving just two commodities and labour. Industry one has no choice of production method: it uses (0.3 units of commodity one, 0.5 units of commodity two and 0.1 units of labour) to produce 1 unit of commodity one. Industry two has a choice between two alternative methods. In the α method (0.6 units of commodity one, 0.2 units of commodity two and 3 units of labour) are used to produce 1 unit of commodity two; in the β method (0.01 units of commodity one, 0.69 units of commodity two and 2.9 units of labour) are used.

Let i be the rate of interest and p and w be, respectively, the price of commodity one and the real wage rate in terms of commodity 2. In a competitive economy, the following must hold for industry one, if wages are paid *ex post*:

$$p = 0.1w + (1 + i)(0.3p + 0.5)$$

The corresponding equation for industry two will, of course, depend on which method is in use. When α is used:

$$1 = 3w + (1 + i)(0.6p + 0.2)$$

but when method β is in use:

$$1 = 2.9w + (1 + i)(0.01p + 0.69)$$

The industry one equation combined with the α (β) equation for industry two defines how the real wage rate, w , falls as the interest rate, i , rises if method α (β) is adopted. At any given i ,

that method - α or β - will be adopted that yields the higher w . It can be shown that for (non-negative) $i < 8.5$ per cent method α will be used, while the β method will be used when 8.5 per cent $< i$. As the real wage 'falls' (and i 'rises') there is a switch from α to β and labour use per unit of output in industry two 'falls' (from 3 to 2.9). Thus direct labour use per unit of gross output in industry two is positively related to the real product wage rate. (See Woods, 1990, § 6.7 for similar examples.)

Example B

We now change to the 'Samuelson-Hicks-Spaventa' type of model in which there are various qualitatively different 'machines', each of which can be combined with labour to produce either the single consumption commodity or new machines of the same kind. We begin with the two-technique example of the following table, in which we adopt Hicks' familiar (α, β, a, b) notation, while W and I represent the maximum wage rate and interest rate, respectively, with a given technique.

Technique	100 α	100 β	a	b	W	I
1	14.400	4.00	0.80	1	1.3158	25 %
2	21.875	1.25	0.75	1	1.1268	33 %

Note that:

- i) $\alpha/\beta > a/b$ for each technique, so that the consumption sector is always more machine-intensive than the corresponding machine sector.
- ii) $W_1 > W_2$
- iii) $I_1 < I_2$

As the real consumption wage rate w 'rises' from zero there is a single switch from technique 2 to technique 1 (at about $i = 10\%$ and $w = 0.72$) and β 'rises' by 220%. Direct labour use per unit of output in the consumption commodity sector is *positively* related to the real consumption wage rate.

Having illustrated the basic point in a simple case, we may now extend this type of model in two ways (simultaneously). First, we may allow each given kind of machine to be combinable with labour in continuously variable proportions, in both consumption commodity and machine production, the former activity always being the more machine intensive: for example, we may introduce two Cobb-Douglas functions. Secondly, we may also allow there to be infinitely many kinds of machine, for example by defining the relevant coefficients to be functions of a continuous variable. As w varies continuously, so does the type of machine in use - and even given the kind of machine, both machine - labour ratios are infinitely variable. Yet it can still be the case that as w 'rises', direct use of labour per unit of consumption commodity output is absolutely *constant*; or 'rises' with w ; or behaves in more complex *non-inverse ways*. This is so when there is no reswitching, no capital-reversing and unlimited substitution possibilities in single-product processes.

Example C

In the examples given above the interest rate was variable; in this and the following examples it will always be constant (and indeed zero in examples D to F). Consider two extremely simple 'Austrian' production processes, each taking two periods to produce one unit of consump-

tion commodity. The 'Greek' process employs (1, 12) units of land and labour, respectively, in the first period and (13, 1) units in the second. The 'Roman' process employs (12, 1) in the first period and (1, 13) in the second. Let r be the real land rental. If the interest rate is $i = 0$, the Greek process is used if $r = 0$ but the Roman process is used if $w = 0$. (The overall land-labour ratio is greater in the Greek process). Suppose now, however, that the interest rate is fixed at $i = 10$ per cent. The Greek process will be used if and only if

$$(1+i)(r+12w) + (13r+w) < (1+i)(12r+w) + (r+13w),$$

i.e. (with $i = 10$ per cent) if and only if $r > w$.

With $i = 10$ per cent, a sufficient 'rise' in (r/w) , from zero, provokes a switch from the Roman to the Greek process, i.e. a 'rise' in the overall land-labour ratio and in the 'final stage' (or second period) land-labour ratio. Notice that, with only two time periods involved, no (r/w) ratio could lead to reswitching with respect to changes in the rate of interest. And yet, with a fixed rate of interest of 10 per cent (or, indeed, any other $i > 9.1$ per cent), the land-labour ratio employed in a competitive economy will be *positively* related to the rent-wage ratio. Indeed, if we may regard the two 'final stages' as constituting 'the' consumption commodity industry, then in that industry direct land use and direct labour use, per unit of output, are *positively* related to the real rent and the real wage, respectively.

Example D

Here we extend the two-technique 'Samuelson-Hicks-Spaventa' model to allow for the use of land (as well as labour) in all four production processes. The familiar Hicksian notation is correspondingly extended to $(\alpha, \beta, \gamma; a, b, c)$ where, of course, γ and c are the amounts of land used per unit of consumption commodity or per machine; in the table, W and R show the maximum wage rate and maximum rent rate.

Tech.	α	β	γ	a	b	c	$100W$	$100R$
I	1	2	1	0.80	1.70	2.30	9.53	8.00
II	1	1	2	0.82	2.32	1.74	7.20	8.57

Note that, for each technique, $(\alpha/\beta) > (a/b)$ and $(\alpha/\gamma) > (a/c)$, so that the consumption sector is unambiguously the more machine-intensive sector. Note too that while $W_I > W_{II}$, $R_I < R_{II}$; so that as the real consumption wage, w , falls from 0.1053 to zero and the real rent rate, r , rises from zero to 0.0857, there is a single switch from technique I to technique II, the rate of interest being constant and zero. (The switch occurs at $w = 0.0163$ and $r = 0.0663$). But that switch of technique, as (w/r) falls, involves a *halving* of β and a *doubling* of γ . Both direct labour use and direct land use, per unit of consumption output, are *positively* related to the corresponding real product factor prices, in a competitive economy with a zero rate of interest. (It is worth noting that if the rate of interest were variable, no ratio of r to w would lead to reswitching with respect to the interest rate - but the $(\Delta\beta/\Delta w) > 0 < (\Delta\gamma/\Delta r)$ result would hold for any *fixed* interest rate in the range $0 \leq i < 11.76\%$).

Example E

As in the previous example, we still have a constant, zero rate of interest and production by means of land, labour and produced inputs. But now there are two alternative processes to pro-

duce commodity one using the same produced input (commodity two), while there are infinitely many ways to combine land and labour to produce commodity two. More specifically, 1 unit of commodity one can be produced by either (4 units of commodity two, 1 units of labour and t units of land), or (1 unit of commodity two, $(1+1)$ units of labour and $(t+2)$ units of land). The unit cost function in industry two is given, in obvious notation, by the very simple Cobb-Douglas function

$$c_2 = \sqrt{wr}$$

Setting $p_1 = 1$ and $p_2 = c_2$, we see that

$$4\sqrt{wr} + 1w + tr = 1 \tag{1}$$

or

$$\sqrt{wr} + (1+1)w + (t+2)r = 1 \tag{2}$$

depending on which method is used in industry one. Now it is easily shown that if $0 \leq r < 0.25w$ or $w < r$ then eq.(1) is the effective real rent/real wage frontier; when $0.25w < r < w$, eq.(2) yields the effective frontier. Of course, at $4r = w$ and $r = w$ entrepreneurs in industry one will be indifferent as to which method of production is used. Let r notionally 'rise' from zero. At first, eq.(1) applies and the direct land-output ratio in industry one is t but for $w < 4r < 4w$ eq.(2) applies and that ratio is $(t+1)$; when $w < r$ however, (1) applies again and thus there is recurrence of the process used in industry one, though there is not reswitching of the complete (economy-wide) technique. (In industry two the land-labour ratio, for example, is always equal to (w/r) so that no process recurrence is possible as (r/w) increases monotonically). Note that, because of the recurrence of the first method in industry one, the direct land-output ratio (labour-output ratio) is at one point *positively* related to the real product rent (product wage) rate. Note also that since the two direct labour-land ratios in industry one, namely $(1/t)$ and $(1 + 1/t + 2)$, can be ranked in either order, the labour-land ratio may *either* first rise and then fall, or first fall and then rise as (r/w) rises.

What of the produced - input coefficient in industry one? As (r/w) rises, that coefficient first falls from 4 to 1 and then rises from 1 to 4. It can be shown that if $(1 > t + 1)$ then (p_2/p_1) is increasing with (r/w) at both switch points; at the second switch, then, the use of input 2 per unit of output 1 is *positively* related to (p_2/p_1) .

Hence there are both process recurrence and positive direct input use/input price relationships in industry one, in the presence of constant returns, a zero rate of interest and unlimited substitution possibilities in industry two. (Of course, recurrence immediately implies non-monotonicity of input use/input price relations).

Example F

Our final example again involves two constant returns to scale industries, production by means of the two commodities, land and labour, and a fixed, zero rate of interest. This time, however, both industries have an unlimited choice of production processes and we focus our attention on the use of commodity one as an input in the production of commodity two. More specifically, the two unit cost functions, each of the 'quadratic square root' type are:

$$2c_1 = (p_2 w)^{1/2} + (wr)^{1/2} + 8(rp_2)^{1/2}$$

$$10c_2 = 2w + (p_1 w)^{1/2} + (wr)^{1/2} + 5(rp_1)^{1/2}$$

Thus each commodity is produced by means of labour, land and 'the other' commodity. The input of commodity one per unit of gross output in industry two, a_{12} , is given by

$$20a_{12} = (w/p_1)^{1/2} + 5(r/p_1)^{1/2}$$

Set $p_1 = c_1$ and $p_2 = c_2 = 1$. At $r = 0$ it is readily checked by substitution that the two 'price = unit cost' equations are solved by ($w = 4, p_1 = 1$) and that, consequently, $a_{12} = (1/10)$. Similarly, at $w = 0$ we have ($r = 1, p_1 = 4$) and hence $a_{12} = (1/8)$. We see, then, that both the 'real product price' of input one (p_1) and the per-unit-of-output use of input one (a_{12}) are greater at $w = 0$ than at $r = 0$. Thus a_{12} is not inversely and monotonically related to p_1 . Note that this is not associated with recurrence of the complete production process in industry two; no such recurrence is possible when there is a smooth, differentiable unit cost function, as here. Recall too that no interest rate effect has been involved.

The examples presented above should, presumably, suffice to show that in a range of different kinds of production model process recurrence is possible and that direct physical input per unit of gross output in an industry can be *positively* related to the corresponding price, even when returns are constant, costs are minimized, and there is no reswitching or capital reversing; in several cases no interest rate effects of any kind were involved.

3. DISCUSSION OF THE EXAMPLES

A central result - arguably the central result - in the theory of cost minimizing choice of inputs, for a given output, in the face of given input prices is that

$$\Delta q \bullet \Delta x \leq 0 \tag{3}$$

where q is the row vector of (present value) input prices of both produced and primary inputs and x is a column vector of input quantities, which minimize costs at those prices. Result (3) is, of course, rather general, covering produced and non-produced inputs, smooth substitution and discrete jumps in inputs, input quantities changing from zero to positive and vice-versa, etc. It is thus important to emphasize that all the examples of Section 2 satisfy eq.(3) in full.

Very often, of course, a statement of eq.(3) is quickly followed by some such sentence as: 'If only one input price changes, so that Δq_j say is the only non-zero element of Δq , then it follows that

$$\Delta x_j / \Delta q_j \leq 0 \tag{4}$$

Each of our examples above both satisfies eq.(3) and violates eq.(4). The explanation, clearly, must be that in our examples *more than one* input price is changing. That is indeed so and we must now state explicitly both the basis underlying our comparative statics examples and our reasons for taking that basis to be superior, from an economist's point of view, to that underlying the move from eq.(3) to eq.(4) (i.e., only one input price is changed).

In each of our examples the assumption is maintained throughout that every industry (both industries when there are two) is always in a long period position, with price equal to unit

cost for the processes actually in use. Each time that we have obtained a result of the generic form $(\Delta x/\Delta q_j) > 0$, the basis of our comparative statics exercise has been that the price change Δq_j is associated with such other price changes as maintain price \leq unit cost for every production process in the economy (with equality for processes actually used). To put it somewhat polemically, our equilibrium comparative statics exercises were based on taking producer equilibrium *seriously*, maintaining it both before and after the input price change in every industry. By contrast, the argument leading to eq.(4) above does *not* take producer equilibrium *seriously*.

It may be helpful to present this point in what may perhaps be the simplest possible form. Consider a one-commodity economy with constant returns and a fixed, zero rate of interest; let the corn, say, be produced by inputs of corn, land and labour subject to the unit cost function $c = c(p,r,w)$, in obvious notation. The corn, land and labour input-output ratios are, of course, given by the three partial derivatives of $c(\cdot)$; say $(a,t,l) = [(\delta c/\delta p), (\delta c/\delta r), (\delta c/\delta w)]$. The usual analysis would then say (correctly) that $[(\delta a/\delta p), (\delta t/\delta r), (\delta l/\delta w)] < 0$. This is indeed true but it is of very limited interest to the *equilibrium* theorist. Let (p,r,w) satisfy $p = c(p,r,w)$, so that the economy is in a long-period position. Then $p < c(p,r+\Delta r,w)$; $p < c(p,r,w+\Delta w)$; and $p + \Delta p > c(p+\Delta p,r,w)$, where each $\Delta q > 0$. Starting 'in equilibrium', one *cannot* change just one input price and *keep* the economy 'in equilibrium'. Hence the (true) statement that $(\delta a/\delta p) < 0$, etc., is not a reliable guide to the signs of $(\delta a/\delta p)$, etc. *when equilibrium is maintained*. That this is so is precisely what our various examples demonstrate. For the economic theorist wishing to examine how input price changes are related to input quantity changes between equilibria, eq.(4) is correct *but irrelevant*. (Of course, eq.(3) is both correct and highly relevant).

The term 'coherent long-period analysis' in the title is, then, intended both positively and critically. Positively, it points to the kind of comparative statics illustrated by our examples, in which the *whole* production system is in equilibrium both before and after the change considered. Critically, it conveys the observation that the familiar analysis of input demand curves embodied in eq.(4) is *incoherent*, in that it implies that the whole production system cannot be in equilibrium both before and after the change considered.

4. CONCLUDING REMARKS

It is not sensible to conduct the long-period comparative statics of input use in a particular industry on the basis of changing one price at a time. For, flukes aside, that will mean that *other* industries are not being kept in a long period position, both before and after the price change - and how can one simultaneously take equilibrium to be very important in the one industry and completely unimportant in all the others? When our comparative statics is done on a coherent basis, however, we find that industry level process recurrence can occur. And that, with or without complete process recurrence, individual input-output coefficients can 'recur' and can be positively related to the price of the input. All these industry level phenomena are possible *without* any system-level 'reswitching'.

ENDNOTES

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