The Investment Multiplier and the Aggregate Rate of Return: A Post-Keynesian View

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ABSTRACT

This paper outlines a Post-Keynesian theory of the 'aggregate rate of return,' which recognizes the change in aggregate profits brought about by the investment multiplier and the income and demand externalities to which it gives rise. The paper analyzes the effect of an increase in investment spending on the aggregate rate of return under three scenarios: 1) imperfect competition; 2) 'pure competition' with diminishing returns; and 3) a full-employment economy where there is an increase in the investment-to-output ratio. Finally, the paper contends that the investment multiplier, which pushes the aggregate return above the private return, comes into play because there is no mechanism for pre-reconciling saving and investment plans before commitments are made.

1. INTRODUCTION

Robert Solow insists that 'the important thing in macroeconomics is not the elementary mechanics of the multiplier, but the reasons for its presence' (1998, p12). Perhaps, but the multiplier's 'mechanics' also deserve attention if only because they are more varied, and more interesting, than one might gather from the familiar textbook accounts. The macroeconomic consequences of an increase in investment expenditure will vary depending on whether markets are competitive, whether firms are operating under increasing or diminishing returns, whether prices exceed marginal costs, and so on. Similarly, the saving necessary to offset additional investment can be brought about by an increase in aggregate income, by a rise in prices, or by combinations thereof.

An examination of some of the multiplier's diverse 'mechanics' may even suggest an answer to Solow's question about 'the reasons for its presence.' Although I consider several variations of the multiplier process below, they all originate from the same circumstance, viz. the lack of a mechanism for pre-reconciling saving and investment plans before decisions are made. In the absence of such a pre-harmonization of intentions, the effect of an increase in investment expenditure is, among other things, to increase aggregate profits, which means that the 'aggregate rate of return' will exceed the private rate of return, or so I shall argue below.

This paper formulates a conception of the 'aggregate rate of return,' which recognizes the change in aggregate profits arising from the income and demand externalities generated by the investment multiplier. More broadly, my aim is to show that the multiplier process is not limited to 'fix-price' economies, nor to the case of imperfect competition, nor even to
economies with unemployment. Rather, the multiplier comes into play whenever *ex ante* saving and *ex ante* investment are unequal, and are brought into balance *ex post* by changes in aggregate income and/or its distribution between wages and profits. The paper is organized in the following way. Part one introduces the concept of the aggregate rate of return, which is a broader notion than the private rate of return, but not as broad the social rate of return. Parts two through four focus upon the aggregate rate of return under imperfect competition, 'pure competition' with diminishing returns, and a full employment economy with an increase in the investment-to-output ratio. And part five draws a few conclusions about the nature of the investment multiplier and its implications for macroeconomic theory.

2. THE AGGREGATE RATE OF RETURN

To begin, let us define the *private* rate of return on investment as the rate of discount that equates the gross profits (sales revenue less variable costs) from a firm's investment project to the capital cost of the project. In equation (1) below, the private rate of return is the value for r, such that the discounted stream of gross profits, \( \Pi_1 + \Pi_2 + \Pi_3 + \ldots + \Pi_n \), is equal to the investment expenditure, I.

\[
I = \frac{\Pi_1}{1 + r} + \frac{\Pi_2}{(1 + r)^2} + \frac{\Pi_3}{(1 + r)^3} + \ldots + \frac{\Pi_n}{(1 + r)^n}
\]

Under certain circumstances, some of which are discussed below, one firm's investment expenditure will increase the sales revenue of other firms. (The investment multiplier is one example of this kind of macroeconomic externality.) Let us define 'aggregate gross profits' as the increase in gross profits accruing to all firms as a result of an investment expenditure. The 'aggregate rate of return' may then be defined as the rate of discount that equates the increase in aggregate gross profits generated by an investment to the capital cost of the project. In equation (2) below, \( \Pi^a \) represents aggregate gross profits, so that the aggregate rate of return is the value for r, such that the discounted stream of aggregate gross profits, \( \Pi^a_1 + \Pi^a_2 + \Pi^a_3 + \ldots + \Pi^a_n \), is equal to the investment expenditure, I.

\[
I = \frac{\Pi^a_1}{1 + r} + \frac{\Pi^a_2}{(1 + r)^2} + \frac{\Pi^a_3}{(1 + r)^3} + \ldots + \frac{\Pi^a_n}{(1 + r)^n}
\]

Although the aggregate rate of return, which may also be called the 'capitalists' rate of return', is a broader concept than the private rate of return, it is not as comprehensive as the social rate of return, which includes all of the costs and benefits associated with an investment expenditure, not just the change in aggregate profits. Moreover, as we shall see later, an investment expenditure undertaken when the aggregate rate of return exceeds the private rate of return will not necessarily bring about an unambiguous improvement in social welfare because an increase in investment expenditure can reduce the real income of large groups within the economy, at least in the short run.

3. THE AGGREGATE RATE OF RETURN UNDER IMPERFECT COMPETITION

Economists sympathetic to the critique by Keynes of the neoclassical theory, but dissatisfied with the microeconomic foundations of *The General Theory*, have attempted to reconstruct Keynesian economics on the foundation of imperfect competition (See Marris, 1991; Mankiw
and Romer, 1991; Dixon and Rankin, 1995; and Solow, 1998). Later I shall argue that the microeconomics of The General Theory are only implausible if you think firms in competitive markets need not forecast their sales revenue before making their supply decisions. In the remainder of this section, however, my aim is to develop a simple equation for the aggregate rate of return to investment under imperfect competition.

When there are unemployed resources, an investment expenditure will generate additional income and spending through the action of the multiplier. To allow for the possibility that an investment expenditure may displace an equivalent, or lesser, amount of investment spending, let us define \( \Delta I \) as the net increase in aggregate investment that is due to an individual firm or public agency’s investment expenditure. If we let \( k \) stand for the investment multiplier, the increase in aggregate income resulting from the investment expenditure will be equal to \( k\Delta I \). This familiar quantity, \( k\Delta I \), may also be characterized as ‘aggregate sales revenue’ because it accrues to firms in total.

To compute the gross profits derived from this revenue, it is necessary to deduct the variable cost of producing the goods that generate the revenue. If we assume imperfect competition as Marris and Solow urge, prices will exceed variable costs by a mark-up. Letting \( e \) stand for the price elasticity of demand for a firm’s output, the profit-maximizing mark-up will be equal to \( e/(e-1) \). Thus, unless a firm believes there has been a change in the elasticity of demand for its product, it has no reason to alter its mark-up. And if the firm’s variable costs are constant or falling, then, assuming no change in elasticity, the firm has no reason to raise its prices either.

For those dissatisfied with the microeconomic foundations of The General Theory, the assumption of imperfect competition is appealing because it provides a simple story line connecting the microeconomic decision making of the firm to the macroeconomics of the multiplier. This nexus may be summed up as follows: given an exogenous increase in demand, the profit-maximizing response for firms in imperfect markets is to increase output rather than prices, provided firms’ variable costs are constant or falling, and there has been no change in the elasticity of demand for their products. Under these conditions, an increase in output can be achieved without a rise in prices or a reduction in the real wage.

We now proceed to derive our equation for the aggregate rate of return. Letting \( m \) stand for the economy-wide, average mark-up, and \( c \) for variable cost, which, for simplicity, I assume to be constant over the relevant range of output, price will be equal to \( cm \), where \( m \geq 1 \). Gross profits as a percentage of sales revenue - the gross profit margin - is equal to \((m-1)/m \). And finally, aggregate gross profits from aggregate sales revenue, represented by \( \Pi^* \) in (3) below, are equal to aggregate sales revenue, \( k\Delta I \), multiplied by the fraction of aggregate sales revenue that is in excess of variable costs, i.e.,

\[
\Pi^* = k\Delta I \frac{m-1}{m}
\]  

(3)

For simplicity, I have expressed aggregate gross profits in a static equilibrium formula. In reality, these profits would accrue to firms over time, and the aggregate rate of return would then be determined by inserting each period’s aggregate gross profits into (2) above.

Although the aggregate rate of return (AROR) captures the basic macroeconomic externalities of the Keynesian paradigm, it bears repeating that the AROR does not encompass other externalities, such as technology spillovers or environmental costs, and thus should not be
mistaken for a welfare concept intended to include all the costs and benefits associated with an investment.

Since aggregate gross profits include the profits of the investing firm, the AROR will exceed the private rate of return (PROR) unless resources are fully employed, so that crowding out is complete and the net change in investment, ΔI, is zero, or the economy-wide mark-up, m, is equal to one, i.e., prices are equal to marginal costs. Some of the implications of these simple results are rather interesting. If we envisage an entire economy operating under conditions of imperfect competition, then prices will exceed marginal costs, there will be excess capacity, and some workers will be unemployed (or at least underemployed). Since these are defining features of imperfect competition, the AROR will necessarily exceed the PROR under imperfect competition. Moreover, in the absence of exceptional circumstances, such as large environmental externalities, it is reasonable to suppose that an increase in investment under conditions of imperfect competition will generate net social benefits. Additional employment will be offered to unemployed and underemployed workers, and aggregate profits will rise. Yet prices and real wages will remain constant, causing no harm to rentiers or workers who are already fully employed.

If there are no externalities other than those generated by the multiplier, we can explain the suboptimal level of aggregate investment under conditions of imperfect competition in the same way the neoclassical economist explains the suboptimal level of investment in pollution-control equipment, that is, by reference to the divergence between the private and social, or, in our case, ‘quasi-social’, rates of return. As long as individual firms in imperfectly competitive markets fail to capture the full benefit of their investment expenditure, the level of aggregate investment will be too low.

Finally, our analysis suggests a shortcoming in the standard method for computing the welfare loss associated with imperfect competition. For the microeconomic price theorist, the social cost of imperfect competition is measured by the so-called ‘Harberger triangle’, which gives the increase in consumers’ welfare when output is increased to the level at which marginal cost is equal to price. It is clear from the foregoing discussion that this technique sharply understates the total welfare loss that is due to imperfect competition. For, if the output of a firm in an imperfectly competitive market were increased, the demand curves of other such firms would shift outward via the income and demand externalities of the multiplier, thereby ‘filling in’, as it were, not one, but many Harberger triangles.

4. THE AGGREGATE RATE OF RETURN UNDER PURE COMPETITION

In The General Theory, Keynes accepted the premises of competitive markets and diminishing returns in order to challenge the orthodoxy on its own grounds. But, according to some friendly critics, these assumptions create intractable problems (see especially Marris, 1991; and Solow, 1998). If firms are operating in competitive markets, then they should be able to sell as much as they wish at prevailing prices. To invoke ‘a felt demand constraint’ to explain unemployment simply contradicts the foregoing premise (Solow, 1998, p3).

Let us grant that the path from increasing returns and monopolistic competition to low-output equilibria is much smoother than Keynes’s attempted route from diminishing returns and competitive markets to unemployment equilibria. That said, it is instructive to ask whether Keynes’s journey is possible at all.
First, we must be careful not to trip over the idea that firms 'can sell as much as they please at prevailing market prices'. If this notion is wedded to the general equilibrium conception in which prices, output, and everything else are determined simultaneously, then we are stymied. But if we insist that firms must forecast prices and demand before making their production decisions, and if actual prices and demand depend on these forecast-driven supply decisions, then a path leading to low-output equilibria comes clearly into view (see Hahn, 1983).

For example, consider the case of a simplified two-sector economy in which firms in sector $Y$ sell only to the workers and owners of firms in sector $Z$ and vice versa (Heller, 1986). Given this premise, firms in each sector must forecast the output of firms in the other sector in order to estimate demand for their own output. The more sector $Y$ is expected to produce, the more its firms will pay out in wages and other factor incomes, and, hence, the greater will be the demand for the output of sector $Z$. It is easy to see there can be both high- and low-output equilibria under these circumstances. If firms in each sector expect firms in the other sector to hire lots of workers, then firms in both sectors will plan for high output and their expectations will be fulfilled. If, however, firms expect low output in the other sector and therefore hire few workers, then their diminished expectations will be fulfilled. This case can be easily generalized to an economy with many sectors where firms, in forecasting demand for their particular output, must implicitly forecast aggregate demand (a circumstance that reveals the macrofoundations of microeconomic decision-making!). Insofar as actual demand depends on such forecast-driven supply decisions, there will be many possible equilibria regardless of whether markets are competitive, or whether firms face diminishing, constant, or increasing returns.

If output is increased under 'pure competition' with diminishing returns, prices must rise because marginal costs are increasing. Keynes did not draw back from this implication, arguing that no one has a right to purchase goods at prices that are low because resources are idle (Keynes, 1936, p.328). Moreover, it should be stressed that, in this scenario, firms are not hiring workers because the real wage has fallen. Rather, prices rise and the real wage falls because the increase in output takes place under conditions of diminishing returns.

To compute the aggregate rate of return when firms are operating under diminishing returns, prices are equal to marginal costs, and some resources are idle, we require two values: (i) the increase in aggregate sales revenue generated by the investment, which is equal to $k\Delta l$; and (ii) the percentage share of this revenue that accrues as gross profits. If, for simplicity, we assume that $Y = L^*$, then the profit share of aggregate income will equal $1-\alpha$, and aggregate gross profits are given by

$$\Pi^* = k\Delta l (1-\alpha)$$

Note that while prices equal marginal costs in this case, since marginal costs are rising, an exogenous increase in investment will increase both aggregate output and prices. Hence, aggregate gross profits, $\Pi^*$, will rise, and the AROR will exceed the PROR.

Unlike the imperfect competition scenario, where an increase in investment generates additional profits and employment with no reduction in the real wage, there are losers as well as winners when investment increases under conditions of pure competition. On one side of the ledger, firms enjoy an increase in profits, and formerly unemployed workers gain employment. On the other side, as prices rise with increasing marginal costs, those workers who were already
employed - the preponderance of all workers - suffer a decline in real wages, at least in the short run. And their odd bedfellows in misery are the rentiers whose fixed income receipts now buy fewer goods.

5. THE AGGREGATE RATE OF RETURN UNDER FULL EMPLOYMENT
Under imperfect competition with constant returns, the extra saving necessary to offset an increase in investment will be provided from the extra income generated by the multiplier. In the case of pure competition with diminishing returns and unemployment, the required saving will be generated, at least in part, by an increase in total income. If we now assume that output is given at full employment, an investment cannot augment aggregate income, hence the necessary saving must come from another source.

Kaldor (1956) developed a model that may be adapted to give the aggregate rate of return under full employment. In introducing this model, Kaldor suggests that the multiplier, which Keynes used to explain the level of output taking the distribution of income as given, can also be used to explain the relation of prices to wages, and thereby the distribution of income between profits and wages, if output is given. Kaldor characterizes his model as 'Keynesian' because it assumes (as we have assumed throughout) that investment is the independent variable. But whereas in The General Theory, saving adjusts to investment through changes in aggregate income, in Kaldor's model, saving adjusts to investment through changes in the distribution of income.

Specifically, Kaldor assumes that with given marginal propensities to save out of profits and wages, the former exceeding the latter, an increase in the investment-to-output ratio will push up prices in relation to wages until the concomitant increase in profits generates the saving necessary to offset the additional investment.

In developing his model, Kaldor builds on the identities: \( Y = W + P \); and \( I = S \), where \( Y \), \( W \), and \( P \) denote income, wages, and profits, and \( I \) and \( S \) denote investment and saving, respectively. Assuming constant saving propensities out of profits and wages, and denoting these saving propensities as \( sp \) and \( sw \), respectively, we have

\[
S = spP + swW, \text{ or } I = spP + swW
\]

which may also be written as \( I = spP + sw(Y-P) \). Dividing by \( Y \) and collecting terms, we obtain

\[
\frac{I}{Y} = (sp-sw) \frac{P}{Y} + sw
\]

and rearranging to find the share of profits in national income, we get

\[
\frac{P}{Y} = \frac{1}{sp-sw} \frac{I}{Y} - \frac{sw}{sp-sw}
\]

The upshot of (6) is that, with given saving propensities out of profits and wages, the share of profits in national income is determined by the share of investment in national output. Since we are interested in the gross profits generated by an investment expenditure rather than the profit share of national income, we may multiply both sides of (6) by \( Y \), which yields:
Because investment is the independent variable, aggregate profits the dependent variable, and $sp$, $sw$, and $Y$ are given, the effect of a change in investment on aggregate profits, $\Pi^*$, may be written as

$$\Pi^* = \frac{1}{sp - sw} \Delta I$$

(8)

It is clear from (8) that aggregate net proceeds will be greater the smaller is the excess of the propensity to save out of profits, $sp$, over the propensity to save out of wages, $sw$.

Under full employment, an increase in investment will increase aggregate gross profits. In this case, as in the case of pure competition, the welfare implications of an increase in investment are ambiguous. In the short run, profits and investment will rise at the expense of wages and consumption. But, in the long run, total output will be higher than it would have been had the additional investment not been undertaken.10

6. CONCLUSION

Table 1 below provides a summary of the three cases discussed above.

<table>
<thead>
<tr>
<th>Table 1: The macroeconomic effects of investment changes under different regimes</th>
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<tbody>
<tr>
<td><strong>Imperfect competition</strong></td>
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<tr>
<td>Cost structure</td>
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<tr>
<td>Resource utilization</td>
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<tr>
<td>Effect of a net increase in investment on aggregate output and prices</td>
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<tr>
<td>Effect of a net increase in investment on the real wage</td>
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<tr>
<td>Aggregate and private rates of return</td>
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<td>Source of additional saving to offset investment</td>
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</table>
In each of the cases summarized in table one, a rise in investment expenditure engenders either an increase in output, or a rise in prices, or some combination of the two. In the case of imperfect competition with constant returns, a firm's investment increases the demand for other firms' goods at prices that exceed variable costs, thereby increasing employment, output, and profits without any change in prices. In the case of pure competition, investment expenditure increases demand, employment, and output, while diminishing returns push up prices and profits. Finally, an increase in the investment-to-output ratio under full employment increases profits as workers bid up prices on the reduced supply of consumption goods.

These scenarios differ from conventional views of saving and investment. In general equilibrium theory, for example, the aggregates of 'saving' and 'investment' give way to a seamless web of contracts stipulating the delivery of 'inputs' and 'outputs' at future dates under precisely specified states of nature. This ideal equilibrium is achieved, in thought at least, by a process of tâtonnement in which no decisions are final, and no trades are permitted, until every 'market' is in balance. From our point of view, the general equilibrium paradigm is useful, not as a model of real-world economies, but insofar as it throws into bold relief the fact that, in actual economies, there is no market in which saving and investment plans can be pre-reconciled before final commitments are made.

To be sure, the New Classical school contends that the market-clearing coherence of general equilibrium theory can be achieved in real-world economies with flexible prices. On this view, the supply of saving and the demand for investible resources are balanced in the market for loanable funds by adjustments in the rate of interest. But this account is open to several objections. In the first place, flexible prices are not sufficient to achieve general equilibrium. Rather, prices must move before any decisions are made on the basis of the 'wrong' prices (Chick, 1983). Second, Greenwald and Stiglitz (1993) have shown that anything short of instantaneous price adjustment may not even be an improvement over the less rapidly adjusting prices of real-world markets. Third, real-world markets lack the full range of future prices necessary for the efficient coordination of saving and investment decisions (Newbery, 1990). And fourth, the rate of interest cannot balance the flows of saving and investment at a level consistent with full employment if it is tethered to the expectations and liquidity preferences of those who hold the massive pre-existing stock of financial assets (Shackle, 1967, pp. 206-09).

The scenarios summarized in table one also diverge from conventional views in that an increase in aggregate investment brings about an increase in aggregate profits. To understand the nature of this relationship, consider an economy with a consumption goods sector and an investment goods sector (Kalecki, 1971). Suppose that wages are the only variable cost in the consumption goods sector, that workers in both sectors spend all their wage earnings on consumption goods, and that entrepreneurs in both sectors buy no consumption goods. Given these premises, it is easy to see that the total sales revenue of the consumption goods sector will be equal to the sum of wages paid in the two sectors. Net revenue, or gross profits, will then be equal to the wages paid in the investment goods sector.

In this simple framework, it is plain that aggregate profits depend on aggregate investment. And while the assumptions are very special, their uniqueness should not obscure the general nature of the mechanism involved. When additional workers are hired to produce investment goods — i.e., goods not available for consumption — an additional money income stream is created with no corresponding increase in the supply of consumption goods. An increase in
the demand for these goods, which will quickly follow unless the entire income stream is saved, can lead to a variety of outcomes three of which were analyzed above. In these scenarios, an increase in demand is met either by an increase in the output of the consumption goods sector at constant prices (the imperfect competition case), or by a rise in prices rather than output (the full employment case), or by a combination of the two (the case of pure competition with diminishing returns). Despite the variety of adjustment patterns, in each case an increase in aggregate investment produces an increase in aggregate profits.

In these scenarios, the ‘action’ unfolds without the benefit of a pre-arranged and mutually consistent scheme of choices. It is in the absence of such a pre-reconciliation of plans that we shall find an answer to Solow’s question about the reasons for the multiplier’s ‘presence’. The notion that decisions may flow from divergent and incompatible expectations goes back to Keynes’s *Treatise on Money* (1930), where changes in the price level originate in the difference between intended saving and intended investment. This idea is, of course, elaborated in *The General Theory*, where changes in aggregate income originate in the difference between planned saving and planned investment. When the decisions of market participants are not mutually consistent, they give rise to income and demand externalities, and, hence, to non-market clearing, sequential processes, which push the aggregate rate of return above the private rate of return, and which comprise, in the broadest sense of the term, the ‘multiplier’.

**Endnotes**

1. City of Seattle Budget Office, 300 Municipal Bldg., Seattle, WA 98104, USA. E-mail: greg.hill@ci.seattle.wa.us. I thank Jerry Allen, Tom Kirn, Glen Lee, Tim Skeel and the anonymous referees for their helpful comments on earlier drafts of this paper.

2. Here ‘underemployment’ refers to the fact that in imperfectly competitive markets, the level of output and employment are suboptimal from a social point of view.


4. This question, I should add, is different from the issue of whether Keynes was right to assume, as an empirical matter, diminishing returns to labour.

5. For a rigorous demonstration of this point, see Farmer (1999).

6. I call this case ‘pure competition’ to dissociate it from the general equilibrium construct.

7. Even though real wages are lower, it does not follow as a matter of logic that the supply of willing workers must therefore decline. There are at least two possibilities where the labour supply will not decline: 1) the marginal employed worker may be receiving some economic rent (Davidson, 1999, p574); and/or 2) the labour supply curve may be backward bending (Robinson, 1947, pp121-22).

8. Additional saving may also come from an increase in the profit share of national income.

9. Note that $sp$ must be greater than $sw$. 

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10. I draw this conclusion assuming 1) that a larger capital stock increases productive capacity and 2) that Kaldor’s full employment assumption holds.

11. In this regard, it is worth stressing Arrow’s point that when individuals must predict what a large number of other decision makers will do, ‘the very concept of rationality is threatened’ (Arrow, 1986, p203).

REFERENCES


