Testing for Unit Roots: Threshold Autoregression and Asymmetric Trend Stationarity

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Abstract

In recent research Enders and Granger (1998) have extended the Augmented Dickey-Fuller statistic to allow the unit root hypothesis to be tested against an alternative of stationarity with asymmetric adjustment. In this paper the threshold autoregressive (TAR) and momentum-threshold autoregressive (MTAR) unit root tests proposed by Enders and Granger are applied to data on UK investment. It is found that the TAR, MTAR and standard Augmented Dickey-Fuller tests fail to reject the unit root hypothesis. In contrast, an alternative modified TAR unit root test is proposed which rejects non-stationarity using specifically derived Monte Carlo critical values. Further testing rejects the null hypothesis of symmetric adjustment, with the form of asymmetry detected supporting recent theoretical predictions for investment behaviour presented by Gale (1996). The results obtained show the form of asymmetry incorporated within the Enders-Granger test to critically influence the resulting inferences drawn

1. Introduction.

IN the examination of macroeconomic time series the Dickey-Fuller (1979) statistic is frequently employed to test for the presence of a unit root. As a result of its observed low power to reject the unit root hypothesis, a number of modifications and extensions of the Dickey-Fuller test have been suggested (see Maddala and Kim 1998 for a survey of these developments). However, it is was not until the study of Enders and Granger (1998) that the implicitly adopted assumption of symmetry underlying the Dickey-Fuller test was relaxed via the use of threshold autoregression (TAR) and momentum-threshold autoregression (MTAR). It is well known that the power of the Dickey-Fuller test depends upon the specification of the alternative hypothesis, and the framework proposed by Enders and Granger (1998) allows the unit root hypothesis to be tested against an alternative of stationarity with asymmetric adjustment. Given the growing literature suggesting that many economic series exhibit asymmetric and non-linear behaviour, the introduction of this realistic alternative hypothesis is of particular interest.

In their seminal study, Enders and Granger (1998) examined and detected asymmetric stationarity in the short-run/long-run interest rate differential. However as this differential is found to be stationary using conventional unit root tests (see, for example, Stock and Watson 1988), the force of the Enders-Granger test was not fully illustrated. More compelling evidence of the practical importance of the Enders-Granger test would be provided if it could be found to reverse conclusions of non-stationarity made using a conventional Dickey-Fuller test. This possibility of 'unit root inference reversal' provides one of the main motivations of the present study. A further feature of this paper will be the modification the Enders and Granger approach via the use of an alternative decision variable in its underlying Heaviside indicator function. While Enders and Granger consider indicator variables based upon the sign and difference of the variable under consideration, this paper will introduce a new test specification based upon the frequency of the data employed.

As this test has not been previously considered, Monte Carlo critical values are derived to allow its valid application. To illustrate the use of the alternative tests, they are applied to data on UK investment expenditure. This series has been selected as the recently proposed theoretical model of Gale (1996) predicts asymmetric investment behaviour. The results derived show that the original Enders-Granger and Dickey-Fuller tests fail to reject the unit root hypothesis, despite the former detecting weak evidence of asymmetry.

Conversely, the new unit root test proposed rejects non-stationarity in favour of asymmetric adjustment about a linear trend. The form of asymmetry detected also supports the predictions of Gale (1996), with adjustment occurring more rapidly when below the equilibrium path defined by the Heaviside indicator function. The results derived show the power of the asymmetric unit root test to depend critically on the form of asymmetry considered.

This paper will proceed as follows. In the following section the Enders-Granger unit root tests will be outlined, with the results of their application to UK investment presented in section 3. Section 4 introduces an alternative specification for the Enders-Granger test. Monte Carlo analysis is undertaken to derive critical values allowing a subsequent application to the UK investment data series. Section 5 concludes.

2. Asymmetric unit root tests

Using the notation of Enders and Granger (1998), the implicitly symmetric Dickey-Fuller test can be considered in its simplest form as:

$$\Delta y_t = \rho y_{t-1} + \varepsilon_t \qquad (1)$$

The sufficient condition for stationarity is $-2 < \rho < 0$. Such a model can obviously be extended by considering, *inter alia*, the inclusion of deterministic terms and lagged values of the dependent variable. To allow for the possibility of stationarity with asymmetric adjustment, Enders and Granger extend (1) via the application of threshold autoregression (TAR), as popularised by Tong (1990). This requires the use of the Heaviside indicator function to partition y_{t-1}. The resulting generalised version of (1) can then be given as:

$$\Delta y_t = I_t \rho_1 y_{t-1} + (1 - I_t)\rho_2 y_{t-1} + \varepsilon_t \qquad (2)$$

where I_t is the zero-one Heaviside indicator function. Enders and Granger consider two specifications for the Heaviside function. Under the first approach, partitioning is based upon the sign of the lagged value y_{t-1}:

$$I_t = \begin{cases} 1 \ \text{if}\ y_{t-1} \geq 0 \\ 0 \ \text{if}\ y_{t-1} < 0 \end{cases} \qquad (3)$$

Under the second approach partitioning is based upon the change in y_{t-1}:

$$I_t = \begin{cases} 1 \ \text{if}\ \Delta y_{t-1} \geq 0 \\ 0 \ \text{if}\ \Delta y_{t-1} < 0 \end{cases} \qquad (4)$$

Models derived using the above rule (4) are referred to as momentum-threshold autoregressive (MTAR) models as the decision rule is dependent upon observed movements in y_{t-1} rather than its level.

Considering a specification such as (2), the sufficient condition for stationarity is $-2 < (\rho_1, \rho_2) < 0$. Convergence to the long-run equilibrium is then given as $\rho_1 y_{t-1}$, when above equilibrium, and $\rho_2 y_{t-1}$ when below. To examine the unit root null hypothesis, the null $\rho_1 = \rho_2 = 0$ is tested. To distinguish between the alternative indicator functions which can be employed, this test is denoted as the Φ test when using (3) and $\Phi*$ when using (4). Given their non-standard distributions, Enders and Granger provide critical values for the implementation of the Φ and $\Phi*$ tests via Monte Carlo experimentation. If non-stationarity is rejected, ρ_1 and ρ_2 converge to multivariate Normal distributions, allowing a further test of the null of symmetry $\rho_1 = \rho_2$ to be applied using a conventional F-statistic.

In the same way that it is possible to consider alternative rules for the Heaviside indicator function, it is also possible to further generalise the analysis by considering alternative attractors defining the equilibrium path. In (2) above the implicit attractor is $y_t = 0$. In practice it may be more relevant to define the attractor as some other constant (e.g. $y_t = \alpha_1$) or as a linear trend (e.g. $y_t = \alpha_1 + \alpha_2 t$). For typically trending macroeconomic time series it is the latter which will be considered the more appropriate option. To permit the use of alternative attractors, the original series y_t can be regressed upon the relevant deterministic terms to derive a new series $\{\tilde{y}_t\}$. Using the derived series $\{\tilde{y}_t\}$ asymmetric unit root tests can be performed, as given by (5):

$$\Delta\tilde{y}_t = I_t\rho_1\tilde{y}_{t-1} + (1-I_t)\rho_2\tilde{y}_{t-1} + \eta_t \qquad (5)$$

In cases where deterministic terms are introduced, the specification of the Heaviside indicator function is modified to use the appropriate revised series. Enders and Granger derive critical values for the Φ and $\Phi*$ tests for the constant and constant & trend cases, which are denoted by the subscripts μ and T, respectively. The testing equation (5) provides the basis for the analysis of UK investment in the following section, with both the TAR and MTAR models of (3) and (4) employed.

3. An application to UK investment

In recent years a number of theoretical studies have emerged depicting asymmetric behaviour in a range of macroeconomic variables. Amongst these are the studies of Ball and Mankiw (1994) for prices and Krane (1994) for inventories. A further example, which provides the basis for the present paper, is the analysis of investment behaviour presented by Gale (1996). In this model endogenous delay is shown to generate cyclical behaviour, with the incentive for delay provided by a payoff externality. As the probability of investment is assumed to be dependent upon the level of economic activity, agents have an incentive to delay investments during a recession. Gale finds this endogenous delay to have an asymmetric effect, lengthening the recovery period, but not the downturn. This analysis supports the predictions of the recent 'option value of waiting' literature which also implies asymmetric investment behaviour (see Dixit, 1992).

It is typically assumed that investment is an $I(1)$ process. However, given the growing literature suggesting that investment adjusts asymmetrically, the unit root hypothesis may be rejected when faced with the alternative hypothesis of asymmetric trend stationarity. To examine this proposition the TAR and MTAR Enders-Granger unit root tests are applied to seasonally adjusted, quarterly data on UK private sector gross domestic fixed capital formation over the period 1962(1) to 1997(4).[2] The results of this analysis are compared to those obtained from an analogous (symmetric) Augmented Dickey-Fuller test.[3]

Following earlier notation, the natural logarithm of the investment series is denoted as $\{y_t\}$. From Figure One it can be seen that $\{y_t\}$ is strongly trending. As a result, $\{y_t\}$ is initially regressed upon a constant and trend prior to applying the Enders-Granger unit root tests to the detrended series $\{y_t\}$. This leads to a test of the form given in (6) below:

$$\Delta\tilde{y}_t = I_t\rho_1\tilde{y}_{t-1} + (1-I_t)\rho_2\tilde{y}_{t-1} + \sum_{i=1}^{p}\gamma_i\Delta\tilde{y}_{t-i} + \upsilon_t \qquad (6)$$

where I_t is determined by (3) and (4) in turn. To examine the impact of the asymmetric stationarity alternative, the results concerning (non-) rejection of the unit root hypothesis are compared to those obtained from a conventional Augmented Dickey-Fuller test as given by (7):

$$\Delta y_t = \alpha + \beta t + \phi y_{t-1} + \sum_{k=1}^{q}\psi_i\Delta y_{t-k} + \zeta_t \qquad (7)$$

Using (7), the unit root hypothesis is examined via the significance of $\hat{\phi}$ using critical values obtained from the response surface analyses of both Cheung and Lai (1995) and MacKinnon (1991). For (6) and (7) the choice of the lag truncation parameters (p, q) was determined to ensure serially uncorrelated residuals on the basis of a fifth order Lagrange Multiplier test. When alternative specifications existed satisfying this criterion the lag truncation parameter was chosen on the basis of the Hannan-Quinn and Schwarz information criteria and the significance of the final lagged difference term.[4] The use of these criteria both resulted in the selection of $p = q = 4$.

Considering Table One, application of the standard ADF test as given in (7) leads to the non-rejection of the unit root hypothesis, with the calculated value of -3.270 against Cheung and Lai (1995) and MacKinnon (1991) critical values of -3.409 and -3.443 respectively at the 5 per cent level of significance. Therefore under the typical symmetric test, investment would be concluded to be an I(1) process. The results of the application of the Enders-Granger Φ_T and Φ_T^* tests are presented in Table Two. Using the TAR model, values of -0.094 and -0.153 are obtained for ρ_1 and ρ_2. Despite an apparent difference in the asymmetric coefficients, the null of non-stationarity can not be rejected against the asymmetric alternative, with the calculated value of $\Phi_T = 5.70$ against an Enders-Granger critical value of 6.25. The results for the MTAR specification are broadly similar. For the MTAR model, values of -0.068 and -0.167 are obtained for ρ_1 and ρ_2, with the calculated value of $\Phi_T^* = 6.48$ against a critical value of 6.78. Therefore although the asymmetric coefficients (ρ_1, ρ_2) suggest the presence of asymmetry, the unit root hypothesis can not be rejected using either of the Enders-Granger tests. However the tests proposed by

Table 1: Augmented Dickey-Fuller test

Statistic	*Calculated value*	*5% Critical value*
$t_{\hat{\phi}}$	-3.270	MK = -3.443 CL = -3.409

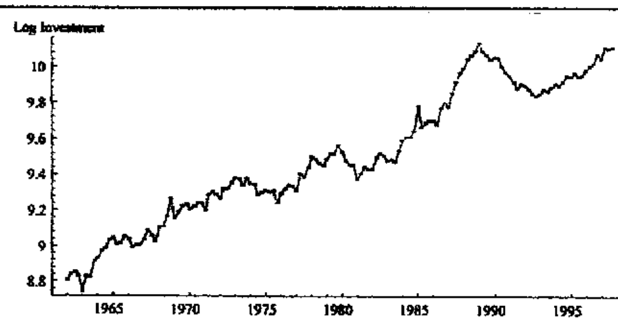

Figure 1: UK Investment

Enders and Granger consider only two partitioning schemes for $\{\tilde{y}_{t-1}\}$, using its sign and its difference. In practice a range of alternative indicators could be employed. In the following section one of these alternatives is considered.

4. A modified Enders-Granger test

Above the Enders-Granger test of (6) has been employed using the Heaviside indicator functions of (3) and (4). However a range of specifications for the Heaviside indicator could be considered, based upon alternative functions of $\{\tilde{y}_t\}$ or different measures of economic activity. One simple specification of the Heaviside indicator function which has an intuitive appeal for quarterly data is given by (8) below:

$$I_t = \begin{cases} 1 \text{ if } \tilde{y}_{t-5} \geq 0 \\ 0 \text{ if } \tilde{y}_{t-5} < 0 \end{cases} \qquad (8)$$

Using this specification $\{\tilde{y}_{t-1}\}$ is partitioned on the basis of the sign of $\{\tilde{y}_{t-5}\}$. Therefore a delay is incorporated in the Heaviside specification, with asymmetry determined according to whether the previous year's value is above or below its defined attractor. The specification of (8) will be combined with the previously used testing equation of (6) to examine the possibility of asymmetric stationarity. However as this specification has not been previously considered the appropriate critical values for its valid application need to be derived via Monte Carlo experimentation. The procedure for obtaining these critical values is outlined in the following subsection.

4.1 Monte Carlo critical values

The relevant critical values for the modified Enders-Granger test were obtained by following the Monte Carlo approach of Enders and Granger (1998). To test the null of a unit root against an alternative of asymmetric adjustment as given by (8), 100,000 replications of the following random walk with drift were generated:

$$y_t = \alpha + y_{t-1} + \omega_t \qquad (9)$$

The $\{\omega_t\}$ series was generated as pseudo *i.i.d.* N(0, 1) random numbers using the RNDNS procedure in the Gauss programming language version 3.2.13. The initial value (y_0) was set equal to zero. The resulting $\{y_t\}$ series was regressed upon a constant and trend and the asymmetric unit root test conducted with partitioning performed according to (8). In the empirical example to follow, the asymmetric unit root test is performed for a sample size of 139 observations. In light of this, the Monte Carlo results were specifically derived for T = 139, after discarding the first 100 observations

Table Two: Enders-Granger unit root tests

	Model	
	TAR	*MTAR*
Asymmetric coefficients		
ρ_1, ρ_2	-0.094, -0.153	-0.068, -0.167
Unit root tests		
ρ_1=ρ_2=0	5.70	6.48
5% critical value[a]	$\Phi_T = 6.25$	$\Phi_T^* = 6.78$

[a] Φ_T and Φ_T^* are linearly interpolated values using the results of Enders and Granger (1998)

to remove the influence of initial conditions. The 1 per cent, 5 per cent and 10 per cent critical values for this modified test are denoted as $\Phi_T^{(4)}$ and are reported in Table Three.

4.2 Empirical results

The results of applying the modified asymmetric unit root test (denoted as $TAR^{(4)}$) to the investment data are presented in Table Three. The values of the asymmetric adjustment coefficients are seen to differ, with $\{\rho_1, \rho_2\}$ given as {-0.059, -0.216}. These results suggest asymmetry to be present. To examine whether this asymmetry is sufficient to reject the unit root hypothesis, the calculated and critical values for the $\Phi_T^{(4)}$ statistic need to be compared. From Table Three it can be seen that the calculated value of $\Phi_T^{(4)} = 8.28$ allows the null of non-stationarity to be rejected at the 10 per cent and 5 per cent levels of significance. The null is also nearly rejected at the 1 per cent level, the calculated value being only slightly below the critical value of 8.36. With the null rejected against the alternative of asymmetric stationarity, the adjustment coefficients $\{\rho_1, \rho_2\}$ follow multivariate Normal distributions. The null of symmetry $\{\rho_1 = \rho_2\}$ can therefore be tested via the conventional F-statistic. From Table Three it can be seen that the null of symmetry is clearly rejected, with a p-value of 0.02 obtained.

Application of the modified Enders-Granger unit root test has therefore led to the rejection of the unit root hypothesis. In contrast, the standard Dickey-Fuller and Enders-Granger TAR and MTAR tests failed to do this, despite the latter tests deriving adjustment coefficients suggesting the presence of asymmetry. The results of the modified test also have a further theoretical appeal as they are in accordance with the predictions of Gale's (1996) model of investment behaviour. From the derived results it can be seen that $|\rho_2| < |\rho_1|$. This result can be interpreted as more rapid adjustment when $\{y_{t-1}\}$ is below its defined attractor. This relates directly to Gale's prediction of sharp declines during the recessionary period in comparison to slow, steady increases during recovery.

5. Conclusion

In this paper the threshold autoregressive and momentum-threshold autoregressive models of Enders and Granger (1998) have been considered. Application of these tests and the standard Dickey-Fuller test to data on UK investment failed to reject the unit root hypothesis.

Table 3: A Modified Enders-Granger unit root test

	$TAR^{(4)}$
Asymmetric coefficients	
ρ_1, ρ_2	-0.059, -0.216
Unit root tests	
$\rho_1 = \rho_2 = 0$	8.28
1%, 5%, 10% critical values[b]	$\Phi_T^{(4)}$= 5.20, 6.18, 8.36
Test of symmetry	
$\rho_1 = \rho_2$	5.497
p-value	0.021

[b]Critical values derived via Monte Carlo experimentation for T = 139.

As the tests presented by Enders and Granger consider only two of a possible range of variables to generate asymmetry, a modified test was introduced employing an alternative Heaviside function. To apply this test, the necessary non-standard critical values were derived via Monte Carlo experimentation. The results obtained using the modified test led to the rejection the null hypotheses of both non-stationarity and symmetry. In contrast to the seminal study where asymmetric stationarity was found in a variable already known to be stationary, the present study led to a reversal of *I(0)/I(1)* inferences obtained under a standard Dickey-Fuller test. This clearly illustrates the empirical importance of the Enders-Granger test. Equally importantly, the results show that the detection of asymmetry under the Enders-Granger test also depends crucially upon the manner in which asymmetry is introduced. The results of the present study can also be seen to support the theoretical arguments of Gale (1996), with more rapid adjustment observed when below the equilibrium path defined by the linear trend attractor.

Endnotes

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2. CSO code DFEB.

3. All empirical analysis undertaken in this paper was performed using Pc-Give 9.0 (see Hendry and Doornik, 1996).

4. See Ng and Perron (1995) for discussion of the properties of alternative methods of selecting the lag truncation parameter.

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