
Uncertainty in a Strong Sense: Meaning and Sources

David Dequech¹

Abstract

*This paper advocates a strong, or radical, notion of uncertainty, in which knowledge, due to the paucity of evidence, is incomplete to an extent that makes it not completely reliable as a guide to conduct. The paper begins by examining how this notion is expressed by Keynes's epistemic approach in his *A Treatise on Probability* and by Davidson's emphasis on nonergodicity. Discussing the relation between epistemology and ontology, I identify some compatibility between these two approaches. Next, I discuss the Expected Utility model and some attempts to generalize it. These attempts go beyond standard EU theory but fail to face the issues of creativity and structural change. I then turn to the issue of the possible gradability of uncertainty in a strong sense. In the final part, before the conclusion, creativity and structural change are highlighted as the most relevant sources of strong uncertainty for economic analyses.*

1. Introduction

This paper advocates a strong, or radical, notion of uncertainty, in which knowledge, due to the paucity of evidence, is incomplete to an extent that makes it not completely reliable as a guide to conduct. This uncertainty does not refer to a situation where the necessary information potentially exists but the agents' computational capability is not

strong enough to perceive it². Conspicuous by its almost complete absence in the mainstream economic literature, this strong notion may be expressed in different forms and under different names in several lines of research. One of these lines is a strand of Post Keynesianism that is defined by its resumption of Keynes's emphasis on uncertainty³. Closer to (different) evolutionary traditions, Langlois's (1984, pp.28-9; 1986, p.228) notion of structural uncertainty (also Langlois and Everett, 1992, pp.69, 73) and Dosi and Egidi's (1991, p.148) notion of strong substantive uncertainty may also be included here. Vercelli's (1991) concept of k-uncertainty (or hard uncertainty, in Vercelli, 1995, 1996), although he points out some similarities with Keynes's view, deserves a separate mention too. In recent years, some works have developed the Expected Utility (EU) theory in ways that may also point to a strong notion of uncertainty. Different conceptions of probability underpin the different ways in which uncertainty in the strong sense suggested above may in principle be expressed. One important distinction is that between the (so-called epistemic) theories of probability in which probability is a property of the way one thinks about the world, a degree of belief, and those theories (sometimes called aleatory) where probability is a property of the real world. Keynes's theory of *A Treatise on Probability* (hereafter *TP*) and the subjective probability theory are

examples of the former, while the frequency theory belongs in the second category. As I argue below, if a property of the real world implies paucity of evidence, this does not necessarily prevent approaches based on an epistemic theory of probability from expressing a notion of uncertainty in the strong sense. Uncertainty may also refer to different things (such as arguments and events) in the different approaches⁴. In principle, I am not particularly in favour of any specific approach, as long as uncertainty is associated with the lack of evidence and the unreliability of knowledge. However, approaches which satisfy this criterion may differ as to another, related issue: the possibility or otherwise of distinguishing degrees of uncertainty in this strong sense. Connected to this are two other questions, to which I also refer below: (a) how to answer to the claim, by some subjective probability theorists, that the distinction between risk (weak uncertainty) and (strong) uncertainty is meaningless? (b) how to deal with problems such as the Ellsberg paradox?

It would obviously be impossible to cover all these approaches here. The next three sections of this paper examine how some of them seem to be able to express a notion of uncertainty rooted in the paucity of evidence. In section 2, I take some controversies around Keynes's *TP* as my starting point, referring also to Keynes's later writings. This discussion is related, in section 3, to Paul Davidson's connection between uncertainty and nonergodicity. Without giving much importance to the *TP*, Davidson is also inspired by Keynes. Section 4 discusses the EU model and some attempts to broaden its scope of application. References to other traditions (especially evolutionary ones) will be spread through the paper. Section 5 turns to the issue of the possible gradability of uncertainty in a strong sense. In section 6, I

present some sources of strong uncertainty that I consider most relevant for economic analyses. A concluding section follows.

2. A Treatise on Probability and Keynes's strong notion of uncertainty

Since the 1980s, with the appearance of several articles and books that explore the connection between Keynes's philosophical and economic writings, his concept of uncertainty itself has become the object of much controversy. Particularly important in this controversy is the *TP* and the issue of whether there is continuity between this work and Keynes's later economic writings, especially *The General Theory* (hereafter *GT*). Also controversial is what continuity means in this case. Those who have argued that Keynes's views on probability underwent significant changes include Bateman (1987, 1990), Davis (1991, 1994a-b), Winslow (1989, 1992) and Gillies and Ietto-Gillies (1991). Reference to the supporters of the continuity thesis, among whom Carabelli (1988) and O'Donnell (1989, 1990a) are perhaps the most vehement, will be made soon. See Gerrard (1992) for an account of some of the main differences between these two lines of argument.

As Keynes's theory of probability in the *TP* has been presented by several scholars, I confine myself to those of its aspects which are more important for my present discussion. Probability in the *TP* is a relation between a proposition *a* and a proposition *h*, where *h* refers to the evidence. The probability represents the rational degree of belief one can have in the proposition *a* given the evidential proposition *h*. Thus, probability is always relative to the evidence. It lies between 0 and 1, that is, between impossibility and certainty⁵. Distinct from probability, and also important in the *TP* approach, is Keynes's notion of weight.

Weight has to do with the evidence on which the probability relation is based. According to Runde (1990), Keynes uses three concepts of weight in the *TP* (*TP*, pp.77, 84, 345), two of which amount to the same thing. Weight represents either the amount of relevant evidence (as opposed to probability, which depends on the balance of favourable and unfavourable evidence) or the evidence's degree of completeness⁶ (the latter is equivalent to the balance of relevant knowledge and relevant ignorance).

I have stated above my interest in the uncertainty involved in economic decisions. Discussions of uncertainty in *TP* terms refer primarily to arguments rather than events and decision outcomes. As noted above, a bridge can be built between the *TP* and the analysis of economic decisions: one can discuss propositions about events. Hoogduin (1987, p.54), for example, suggests that expectations (which are at the centre of Keynes's theory of decision making in *The General Theory*) 'may be interpreted as propositions about future events'.

According to Lawson (1985, p.913) and Runde (1990, p.284), Keynes does not provide an explicit concept of uncertainty in the *TP*, while Winslow (1992, p.107) believes the contrary. At any rate, some authors use the *TP* framework to define uncertainty. There is some controversy in this regard. For Lawson (1988, p.48; also 1985), uncertainty 'corresponds to a situation in which probabilities are not numerically determinate - or even comparable, in terms of more or less, with other probability relations'. Rotheim (1988, p.88), Brown-Collier and Bausor (1988, pp.238-9), Hamouda and Smithin (1988, p.160) and Dutt and Amadeo (1990, pp.105-6) agree (see also Carabelli, 1988, p.47, and Arestis, 1992, p.91). However, Hoogduin (1987, p.54n) maintains that

numerical indeterminacy and noncomparability of probabilities are 'only possible (non-necessary) characteristics of one of the two dimensions of Keynesian uncertainty' (see also Kregel, 1987). This first dimension relates to probability, while in the second dimension, according to Hoogduin's interpretation, weight is a measure of uncertainty; uncertainty means low weight (also O'Donnell, 1991a, p.19; 1990b). Similarly, Runde (1991, pp.131, 133) argues that in the *TP* uncertainty does not mean only a complete absence of probable knowledge, referring also to a situation in which there is some sort of probable knowledge but the argument has little weight (see *TP*, p.342, Runde, 1990, p.290; Meeks, 1991, p.153; Gerrard, 1995, p.184). In Runde's (1990, 1991) view, uncertainty in this case refers to the reliability of probable knowledge as a guide to conduct in practice.

My own view is that in the *TP* framework, uncertainty in a strong sense is related both to non-numerical probabilities, in the *TP* sense, and to low weight (in frameworks other than the *TP*'s, numerical probabilities may be associated with strong uncertainty, as I discuss in section 4)⁷.

Within the *TP* framework, the absence of numerical probabilities could qualify for an association with radical uncertainty, in the sense that it results from want of information (instead of insight, as in the case of unknown probabilities). Keynes asserted in the *TP*: 'The problem [of non-numerical probabilities] does not concern the case that the method of calculation prescribed by theory is beyond our powers or too laborious for actual application. No method of calculation, however impracticable, has been suggested' (p. 32; see also Carabelli, 1988, p.43; Lawson, 1988, p.43). This could qualify for an association with Keynes's famous quote in his 1937 *QJE*

article, in which he states that about uncertain matters 'there is no scientific basis on which to form any calculable probability whatever. We simply do not know' (*CW XIV*, p.114)⁸.

It seems that the cause of numerical indeterminacy, in Lawson's (1985, p.914) interpretation of the *TP*, is the lack of relevant evidence. More precisely, numerical probabilities in the *TP*, as in some other theories, would require the existence of 'identifiable classes of homogenous or repeated events that are in reasonable agreement with the mathematical concept of independently repeated random events' (Runde, 1995b, p.336) - see Runde (1994b) for a detailed discussion of the *TP* on this. Lack of evidence is also what is behind low weight, so that the two are related. Hoogduin (1987, p.58) argues that 'in practice an argument with a very low weight will often have a non-numerically measurable probability [in the *TP* sense], although this is not necessarily so'. Low weight does not necessarily imply the absence of numerical probabilities, but, even when probable knowledge in the *TP* sense is possible, low weight implies uncertainty as to the reliability of knowledge as a guide to conduct. By contrast, weak uncertainty would refer, in the *TP* terms, to the presence of numerical less-than-unity probabilities and maximum weight.

3. Davidson's emphasis on nonergodicity and Keynes's strong notion of uncertainty

Davidson (1988, 1991a) has borrowed modern terminology from the theory of stochastic processes to express Keynes's notion of uncertainty in terms of nonergodicity. In Davidson's conceptualization, probability seems to be a property of the real world⁹. For those who propose that we interpret Keynes's mature economic work in terms of the *TP* framework, this is not a proper way of presenting Keynes's position, since Keynes

defended in the *TP* an epistemic theory of probability.

In my view, however, the difference between Davidson's approach and one based on the *TP*, in what refers to the roots of a strong concept of uncertainty, is not as significant as it is argued or implied by some of those who have defended the continuity thesis (there is a potential difference in another respect, which I discuss below). The discussion of uncertainty always has both an epistemological and an ontological content. The notion of uncertainty is always epistemological in the sense that it is associated with the lack of some kind of knowledge (certain or fully reliable probabilistic knowledge), and knowledge is the subject matter of epistemology; at the same time, the notion of uncertainty always has an associated view of reality, and therefore has an ontological counterpart, given that ontology refers to the study of the nature of reality. Nonergodicity refers to an *ontological* characterization of the nature of reality and it is compatible with different conceptions of probability. Nonergodicity pertains, therefore, to the ontological side of the definition of uncertainty. This characterization of reality has implications in terms of what kind of knowledge people may or may not have of reality, and therefore it is also connected to epistemology. In the previous section, I referred to the absence of numerical probabilities and low weight as situations in the *TP* whose origins can be located in the paucity of evidence. Nonergodicity is associated with the possibility of structural changes and this possibility is a reason why the evidence available to economic decision makers is scant and knowledge not entirely reliable. It is not the only reason for the paucity of evidence and the unreliability of knowledge¹⁰, but I believe it is, directly and indirectly, the most

fundamental one in the most relevant economic decisions (this should become clearer with the discussion of the sources of uncertainty in section 6). Some examples used by Keynes himself to clarify his notion of uncertainty in his later writings are connected with structural change (and thus with nonergodicity), as shown below. The possibility of structural change is a property of the real world, but it can - and must - be dealt with by those who favour an epistemic approach such as the *TP*'s. My suggestion in this regard is that the possibility of structural change affects weight, by implying incompleteness of evidence - the possibility of structural change may be seen as sufficient but not necessary for low weight. I am arguing, thus, that the reliability, as a guide to conduct, of probability as a feature of the way one thinks about the world is affected by the features of the world. Therefore, another important point to make is that, even if one uses an epistemic theory of probability, this very important source of strong uncertainty lies in the external reality (see Carvalho, 1988, p.78, for whom uncertainty 'is not simply a result of defective methods of reasoning. The insufficiency of premises is rooted in objective features of actual social processes'; see also Dow, 1995, p.118). This is valid even if low weight is associated with some probable knowledge in the *TP* sense. It should be noted also that Keynes's epistemic approach is accompanied, in the *TP*, by a view of the ontological requirements for a reliable use of the past as a guide to the future, and, especially in his mature writings, by an ontological conception of economic reality as organic (see Brown-Collier, 1985, Hamouda and Smithin, 1988 and Rotheim, 1989-90; for a different position, see Davis, 1989-90).

Davidson's position, however, may

significantly differ from one based on the *TP* regarding the gradability issue, discussed in section 5.

4. The expected utility approach and a strong notion of uncertainty

There are two main reasons to discuss the EU approach here. First, subjective versions of this approach reject at least one type of distinction between weak and strong notions of uncertainty (or between risk and uncertainty). Second, some recent attempts to generalize the EU approach seem to allow room for such a distinction and to point to an incorporation of uncertainty in a strong sense. Let me start with strict EU theory.

4.1 Expected utility theory in strict terms

The origins of the EU model go back to Bernoulli's solution to the St. Petersburg paradox. Von Neumann and Morgenstern provided a first axiomatization, based on the frequency theory of probability. For my purposes in this section, it is more interesting to concentrate on Subjective Expected Utility (SEU) theory, as represented by Savage's (1954) version, since it incorporates (and indeed is a major contribution to) the subjective probability approach. Why is it more interesting? Several authors associate a strong notion of uncertainty with the absence of numerical probabilities, in the sense of Keynes's *A Treatise of Probability*. Similarly, Davidson also contraposes uncertainty and probability (even though his notion of nonergodicity is compatible with different conceptions of probability). Subjective (or personal) probability theory, typically represented by the Bayesian approach, argues that it is possible to assign numerical probabilities to virtually any proposition or event, even, as Shoemaker (1982, p.536) notes, to unique events. 'Subjective

probabilities are not necessarily based upon much or indeed any evidence' (Kelsey and Quiggin, 1992, p.135). Thus, the often-made association between uncertainty and the absence of measurable probabilities would not make sense. The same would apply to a distinction between risk (or weak uncertainty) and (strong) uncertainty on these grounds¹¹. However, strict Bayesianism is more restrictive than it may appear.

Subjective probability theory was originally developed by Ramsey and de Finetti and later by Savage. Betting rates are the mechanism through which subjective probabilities can be measured, allegedly also in situations that would be otherwise characterized as ones of Knightian or Keynesian uncertainty. The approach does not imply that probabilities can be assigned in any manner. Savage (1954, p.3) does 'not deny that two reasonable individuals faced with the same evidence may have different degrees of confidence in the truth of the same proposition', but rationality requires consistency. Consistency, in its turn, requires that the agent cannot be induced by a clever bookmaker to a series of bets that implies loss whatever the final results, that is, consistency prevents Dutch books. This also requires that agents revise their probabilities by updating them according to Bayes' rule as new information is acquired (Kelsey and Quiggin, 1992, p.144). Probabilities are attached to states of the world. The decision maker chooses among acts the consequences of which depend on which state of the world prevails¹². States of the world are defined independently of acts.

4.2 Strict EU theory and strong uncertainty

It is recognized even within mainstream economics that EU theory, in its strict versions, has been subject to several important challenges (Shoemaker, 1982 and Machina, 1987 provide useful surveys). Some of these

challenges are posed to the standard theory even when objective probabilities are available (for example, when these probabilities are presented to subjects in empirical experiments), in phenomena such as the Allais paradox, preference reversal and framing effects (on these challenges, see also the survey by French and Xie, 1994). Even more important from my perspective here are two other challenges, particularly directed to strict Bayesianism. The first, as Runde (1995b, p.340) observes, comes from the fact that betting is much less widespread than the theory would make us expect. The second comes from the Ellsberg paradox. It is more famous, having even been included in important graduate textbooks (e.g. Varian, 1992, pp.193-4; Kreps, 1990, pp.116-7). Ellsberg's (1961) essential finding was that subjects are sensitive to more than probabilities and may have uncertainty (or ambiguity, to use another common term) aversion. The Ellsberg paradox violates Savage's sure thing principle¹³ and the lack of widespread betting runs counter to the complete ordering axiom. These two challenges are associated with the presence of uncertainty in a strong sense.

The Ellsberg paradox can be related to Keynes's notions of weight and confidence in the *TP* and the *GT* respectively (Frisch and Baron, 1988, p.152; Camerer and Weber, 1992, p.327, 331; Anand, 1993, p.113-4; Runde, 1995b, p.343; Carabelli, 1996, p.3), whereas the distinction between belief and confidence is alien to strict Bayesianism. Ambiguity results from missing information (Frisch and Baron, 1988, p.152; Camerer and Weber, 1992, p.330) and therefore is inversely related to weight and reliability. As seen above, weight in the sense of the evidence's degree of completeness affects the reliability of knowledge as a guide to conduct. The relation between weight and confidence in

Keynes is somewhat controversial, but at least arguments of greater weight (in the above sense) can be said to allow for more confidence (Dequech, 1997b).

Confidence - or the lack thereof - also underlies Keynes's notion of liquidity preference (Runde, 1994a, p.141; Dequech, 1997b) and this notion is related to what is, in SEU terms, people's refusal to bet. Since agents who prefer liquidity are indefinitely postponing their action, they are not revealing any propensity to act and therefore their subjective probability cannot be assessed, contrary to the strict Bayesian idea that this is always possible. Davidson (1991a, p.134), referring to Hicks (1979, p.113n), notes that Keynes's liquidity preference is associated with a violation, under uncertainty, of the complete ordering axiom.

Thus, in SEU terms, there is uncertainty in a strong sense when a person is uncertain about probabilities¹⁴. The person can be even reluctant to bet (and if we want to relate this to Keynes's liquidity preference, this reluctance must be due not merely to fear of asymmetric information, but also to missing information). A situation in which a person does not know which event will happen but unambiguously knows the probability of each and every event involves risk but not strong uncertainty¹⁵. The same applies to the case where the ambiguity over probability can be expressed as a second-order probability. In this case, a person will conceive a set of probability distributions rather than only one, but will be able to unambiguously assign probabilities to each of these distributions (Camerer and Weber, 1992, p.331).

As the discussion related to the Ellsberg paradox shows, nonergodicity may be sufficient but is not necessary for a notion of uncertainty that goes beyond standard EU theory. On the other hand, Ellsberg-type

problems are quite artificial, and the most relevant economic situations of strong uncertainty do, in my view, directly or indirectly involve nonergodicity.

Finally, it should be noted that uncertainty in a strong sense may prevent the very use of the notion of state of the world as an event independent of acts. A less restrictive notion of event, as something that can be endogenous to the decision process, is needed to allow for creativity. Technical change, for example, implies endogeneity of events (Dosi and Egidi, 1991, p.148).

In short, strong uncertainty is contrary to standard EU theory, for the latter requires, to use Hamouda and Rowley's (1987, p.47) terms, simplicity, completeness and invariance (also Dosi and Egidi, 1991, pp.147-8; Vercelli, 1996, p.5). Without this, a unique, additive and fully reliable probability distribution cannot be conceived.

4.3. Generalizations of EU theory and strong uncertainty

In response to the various challenges mentioned above, a sizeable literature has appeared that attempts to generalize the EU approach, not necessarily in Bayesian terms. As some of the challenges to the standard model may arise from strong uncertainty, it is not completely surprising if lines of research within this literature point to an incorporation of strong uncertainty (for a similar opinion, see Runde, 1995b, Vercelli, 1995, 1996, and Carabelli, 1996). Let me briefly refer to two such lines of research, without any intention to be comprehensive (for a more detailed discussion of these and other alternatives to standard EU and particularly SEU theory, see Kelsey and Quiggin, 1992, and Camerer and Weber, 1992; see also table 1 in Vercelli, 1996).

(a) The multiple-priors approach

This approach abandons the standard idea that agents have a unique probability distribution. Ellsberg (1961, p.661) himself introduces a set of probability distributions and also refers to the confidence that the decision maker has in his/her estimates. Thus, the idea of full reliability is also abandoned, which explains the paradox, since in Ellsberg's experiments people prefer more reliable information. A similar approach, in an otherwise Bayesian framework, is pursued by Gärdenfors and Sahlin (1982). What distinguishes them from strict Bayesianism is their consideration of the 'epistemic reliability' of the available information about possible states and outcomes. They 'use a class of probability measures instead of only one' and 'add a new measure which ascribes to each of these probabilities measures a degree of epistemic reliability' (p. 362-3). Gärdenfors and Sahlin (1982, p.368) themselves relate their notion of epistemic reliability to weight in the *TP*, although 'it is difficult to say how far this parallel can be drawn' (see also Frisch and Baron, 1988, pp.151-2). An axiomatization of the multiple prior approach is provided by Gilboa and Schmeidler (1989).

(b) The nonadditive prior approach

This type of model, again axiomatically developed by Schmeidler (1989) and Gilboa (1987), replaces the Bayesian prior with a nonadditive measure or capacity. This results from the introduction of weaker axioms than Savage's sure thing principle and allows for an explanation of the Ellsberg paradox. A nonadditive measure may exhibit uncertainty aversion. The degree of subadditivity may be taken to represent one's confidence in probability assessments (Karni and Schmeidler, 1991, p.1803). Thus, Camerer and Weber (1992, p.348) clearly associate nonadditivity with Keynes's weight of

evidence and measure weight, or faith in the likelihood of events, via the degree of nonadditivity¹⁶.

In these generalizations of EU theory, to the extent that they accommodate strong uncertainty, the latter is not associated with the absence of numerical probabilities, for numerical probabilities may exist but are not necessarily unique, additive or totally reliable¹⁷.

According to Vercelli (1995, pp.252, 259), these generalizations of EU theory are still static and 'do not have much to say about decisions involving the passage of time'. In addition, more debate is needed to assess the proximity between these generalizations and the strong notions of uncertainty that refer more explicitly to structural change. For example, these generalizations are able to deal with Ellsberg-type problems, where people do not have enough information on the probabilities of all states of the world but the set of all possible states of the world is known. These problems have an important characteristic that Bausor (1985, p.74) identifies in the situations mainstream economics focuses on, namely: "Learning" never generates new states of the world, but affects only confidence in which (pre-existing) state is true'. In contrast, when structural change and especially creativity are possible, acts create new, previously unimagined and unimaginable events, so that the notion of state of the world as something independent of acts does not even apply. The attempts to generalize EU theory must face this issue.

5.Degrees of uncertainty?

I have so far discussed how some approaches to uncertainty are or seem to be able to express it in a strong sense. I now turn to the following question: is this uncertainty gradable? Can one speak of degrees of uncertainty, at least in *ordinal* even if not in

cardinal terms? While a *TP*-based approach, as well as some generalizations of the EU model, would provide a positive answer, Davidson's position is not totally clear. Thus, even though I pointed out in section 3 that the difference between Davidson's view and those based on the *TP* is not as significant as it might seem, in what regards the roots of a strong concept of uncertainty, now a potential difference appears.

A good starting point for this discussion is the exchange between Davidson and Runde in the *Critical Review*. Runde (1993) criticizes what he identifies as Davidson's (1989) adoption of two incompatible positions on uncertainty. On the one hand, according to Runde, the 'official' Davidson establishes an epistemological dichotomy between knowledge and uncertainty (the absence of knowledge) and a corresponding ontological dichotomy between ergodicity and nonergodicity. Runde (1993, p.385) argues that these dichotomies create a serious problem: 'If nonergodicity implies ignorance about the future, then people have no basis on which to adjudge some thing as more or less uncertain than others'. This would make no particular course of action more attractive than others. On the other hand, Runde continues, the 'unofficial' Davidson uses the notion of sensible expectations, which are based on the existence of social institutions, to describe how agents cope with uncertainty. 'Davidson's "sensible expectations" have no place on either pole of his knowledge/uncertainty opposition'. Beliefs based on the existence of social institutions are 'arrived at in a "nonergodic environment" and therefore do not qualify as knowledge on Davidson's official account' (*idem, ibidem*). Therefore, Runde concludes, Davidson's notion of sensible expectations breaks with the knowledge/uncertainty dichotomy and

implicitly relies on a third category.

It is here that a connection with the previous discussion appears. Keynes's epistemic theory in the *TP* would provide such a category (while not using the ergodic/nonergodic dichotomy)¹⁸. In the *TP* (p. 342), Keynes refers to a situation in which it is possible to have probable knowledge based on little evidence. As mentioned above, Runde (1991) sees uncertainty in this case as characterized not by a complete absence of probable knowledge but by low weight. Having weight as its measure¹⁹, uncertainty would then be gradable²⁰.

In his response to Runde and others, Davidson (1993, p.431) denies that he takes two incompatible positions, but his view on this issue is still not completely clear to me. On some occasions he denies the gradability of uncertainty (1993, pp.431-2), while on others he seems to accept it (1991a, p.142; 1991b, p.75). In my view, there is still the need to further explore and clarify the different positions with regard to this issue, which lies beyond the scope of this paper.

At any rate, it is my view that even those who argue, based on the *TP*, that it is possible to have some kind of probable knowledge under uncertainty²¹ must recognize that such knowledge is incomplete and not fully reliable as a guide to conduct²². This is so because it is accepted that in this case uncertainty is marked by low weight, which implies a significant degree of ignorance. Considering this, it is not so important, then, whether it is possible or not to have probable knowledge under uncertainty: factors such as what I call an optimistic disposition to face uncertainty must influence action, even if expectations are based on probable knowledge (in the *TP* sense). When probable knowledge is not possible, this optimistic disposition - which is not exactly the same as animal spirits - affects

not only confidence but also (together with creativity) expectations themselves (see also Carvalho, 1988, p.77n). An extended presentation of my own view on the determinants of the state of expectation appears in Dequech (1997b), but the following comments suffice for my purposes here. The state of expectation depends on expectations themselves and on the confidence in them (Keynes, 1936, p.148). Expectations are determined by three factors: knowledge, the optimistic disposition to face uncertainty (via spontaneous optimism), and creativity. Confidence depends on how much uncertainty a person perceives and how willing the person is to face or to avoid this uncertainty. Confidence is then a combined result of what I call uncertainty perception and uncertainty aversion. While uncertainty aversion is a result of the optimistic disposition only, part of uncertainty perception may have a more concrete basis in knowledge and thus may be independent of that disposition. The relation between the optimistic disposition and uncertainty perception is a difficult one to establish, but the former can be seen as also affecting the latter (see also Dow, 1995).

Those who propose that we measure uncertainty via weight may have in mind a cardinal measure, while I think that in many economic situations we should be contented with ordinal comparisons of degrees of uncertainty. Furthermore, weight may not provide an incontestable measure of uncertainty, since there is disagreement among Keynes's interpreters as to the objective or otherwise character of weight²³. If weight is to measure uncertainty (actually, a complement of uncertainty), then the notion of uncertainty becomes affected by the same problem involved in defining weight as the evidence's degree of completeness: how can we, under uncertainty, know how ignorant we are? I agree with Runde (1990, p.282) that 'we do

often know of, or at least are able to identify, factors of which we are to a large extent ignorant'. This is easiest to see in cases of ambiguity, as in the Ellsberg paradox, where complete information potentially exists and is just not made available to the decision-maker. Even so, if uncertainty implies, in its strongest form, that some information will only be created in the future, our ignorance cannot be defined precisely in objective terms *ex ante*. In these cases, I would argue that, as Runde himself (1990, p.283) admits, 'we can never say how complete our information is at any point'.

It should be noted that, although uncertainty perception has a more concrete basis on people's knowledge of economic reality and does not depend solely on people's optimistic disposition (Dequech, 1997b), this knowledge is not totally objective. One's theory of economic reality is crucial for one's assessment of uncertainty (this is clearly seen in the case of weight as the balance of relevant knowledge and relevant ignorance, which, as noted above, is equivalent to weight as the evidence's degree of completeness); as people, including economists themselves, disagree on what is the best theory, they will have different assessments of uncertainty. Indeed, once it is recognized that the assessment of uncertainty depends on the specific theory adopted, people may be uncertain about their own assessment, so that a higher order of uncertainty appears as to the appropriateness of the theory on which their assessment is based, as suggested by Dow (1995, p.124). This is uncertainty about uncertainty. This higher-order estimation (or an even higher-order one entailed by it) is not totally objective either.

Followers of the *TP*-based and of the Davidson approaches could agree on the definition of uncertainty in a strong sense as the characteristic of a situation in which

knowledge is incomplete and not totally reliable. The gradability or otherwise of uncertainty depends on the notion of knowledge. If knowledge is restricted to knowledge of completely reliable probability distributions, then uncertainty cannot be gradable. Knowledge would be absent, more than incomplete, and reliability would not be an issue. Reliability could not then be used to contradict the strict SEU claim that the distinction between strong and weak uncertainty (or risk) is meaningless, nor to address the Ellsberg paradox. If knowledge is not restricted to that, then one can have some knowledge or, equivalently, knowledge can have some reliability, under uncertainty²⁴. To accept this point one does not necessarily have to adopt the *TP* theory of probability - as exemplified by Vercelli's notion of gradable 'k-uncertainty'²⁵ and by some generalizations of Expected Utility theory, in which reliability (or ambiguity) would provide a criterion to make uncertainty gradable, as in the *TP*.

For me, uncertainty does not mean complete ignorance. First of all, decision-makers are or at least may be aware of uncertainty. In Hicks's (1977, p.vii) aphorism, people [may] know they don't know (if they actually realize that depends on the theory they adopt or on the advisors they employ; for example, we should not forget that neoclassicism has been the most prestigious school of economic thought and that its influence may lead academics and non-academics to fail to perceive uncertainty).

Second, my conception of reality is such that there is more to ontology than just the previous reference to nonergodicity, and this provides an ontological basis for knowledge in a nonergodic environment. One of the aspects of reality about which some knowledge is possible is the existence of

institutions that reduce uncertainty. Foremost among these institutions are contracts. Other examples may include supply and trade agreements and market makers (see Dequech, 1997a). Furthermore, in a perhaps rockier terrain, Lawson (1985, pp.916-7, italics omitted) maintains that 'people have an extensive knowledge of social practices' such as conventions²⁶, and 'an important way (although not the only way) in which such knowledge is obtained is (...) through their own participation. Indeed, to use such knowledge in order to "get by" is to help constitute those very practices' (also p. 926). The knowledge of social practices (or institutions, in a broad sense) also appears in the institutionalist literature. Uncertainty can be gradable as one can have knowledge of more aspects of reality in some situations than in others. One should, however, distinguish between the knowledge of how institutions and conventions have worked so far and the knowledge of how they will work in the future. The potential for unpredictable social change must be admitted. At any rate, it seems that we do have, even in a nonergodic environment, some type of knowledge (which is not knowledge of completely reliable probability distributions) about the working of the social world that allows us to believe in the stability of at least some social practices. This belief is not totally unfounded; on the other hand, this knowledge is incomplete and not fully reliable as a guide to action²⁷.

6. The main sources of economic uncertainty in a strong sense

The sources of strong uncertainty discussed below are not the only ones, but the ones I tend to consider as ultimately the most important ones in economic decisions regarding investment, production, liquidity preference, etc.. First of all, the economic

future is uncertain because it depends on (broadly) historical factors. Unpredictable historical changes can occur. 'Societal actions can permanently alter economic prospects' (Davidson, 1991a, p.132n). A major aspect or example of historical change is the change in knowledge. This is a good example to start with because future knowledge is not knowable in advance, by definition²⁸. The fact that the future is affected by our knowledge and that we cannot know now what is going to be known later is a crucial source of uncertainty²⁹.

This leads to the issue of technological or managerial innovations, which consist of the application or the very materialization of new knowledge. Schumpeter's (1943) notion of creative destruction comes to the fore. Davidson (1982-83, pp.192-3), for example, resorts to the Schumpeterian entrepreneur to explain the Post Keynesian idea that the future is a result of '*human creativity*', while other theories, such as one based on the rational expectations hypothesis, 'restrict entrepreneurship to robot decision-making' (original emphasis). Similarly, Rutherford (1984, p.381) associates Keynes's notion of uncertainty with the possibility of unpredictable structural changes³⁰ and establishes a connection between this and Schumpeter by referring to a quotation from Schumpeter on innovation in Keynes's *Treatise on Money* - see also Feltz and Hoogduin (1988, p.114), Dow and Dow (1985, p.55), Lavoie (1992, p.44) and Deprez (1985-86, p.258). Keynes himself (*CW* XIV, pp.113-4) mentions 'the obsolescence of a new invention' as an example of uncertain matter about which 'we simply do not know'³¹. Innovations are particularly important in this context because competition, in a capitalist system, stimulates decision makers to innovate in search for extra profits, so that there is an endogenous pressure for

something that causes uncertainty (see Kregel, 1990, p.90, on Shackle).

Furthermore, historical changes can be of a more typically political or cultural nature. They have a significant impact on preferences, work relations, the workers' bargaining power, government decisions, etc.. Again, Keynes's 1937 examples of uncertain matters help to clarify his notion of uncertainty, for they include 'the prospect of an European war' and 'the position of private wealth owners in the social system' some thirty years from now³².

It is important to clear up the role played by the interdependence among economic agents in creating uncertainty. The result of one's decision depends on the decisions taken by others. Interdependence is not a source of uncertainty in the sense that it creates complexity in a constant environment (for this would lead to the less radical problem of insufficient capability on the agent's part). First, one has to consider organic interdependence, where the whole is not the mere sum of the parts (see Rotheim, 1995). Second, organic interdependence has to be combined with the possibility of creative individual behaviour. Finally, interdependence creates uncertainty in the sense that expectations must be about other people's expectations - they are 'thoughts about thoughts', in Shackle's (1972, p.71) phrase - and this spreads uncertainty. There is uncertainty about expectations³³, and interdependence spreads this to practically all expectations. Thus, even if long-term expectations are the ones most affected by the prospect of innovation or major social changes, this should not prevent one from acknowledging that uncertainty is extremely pervasive. Finally, the idea that any economic future, near or remote, is uncertain is fundamentally connected with the principle of the autonomy of expenditure³⁴, which is the

basis for Keynes's refutation of Say's Law³⁵. They belong to the same - and basic - theoretical level, in the sense that they presuppose one another. Economic agents can never know in advance nor estimate with a fully reliable probabilistic calculus the proceeds they will obtain from investment, production, or, more generally, from their portfolio of assets. On the other hand, it is uncertainty that makes those who have money indefinitely defer the decision to buy goods or less liquid assets. This is why in my opinion economic uncertainty and the principle of the autonomy of expenditure presuppose one another.

7. Conclusion

This paper has defended a strong concept of uncertainty, in which the paucity of evidence makes knowledge incomplete and not totally reliable as a guide to conduct. Different approaches can express this notion. They resort to different conceptions of probability and attach probability to different things. Even if the probability theory underlying the concept is one in which probability is a property of knowledge, the main source of uncertainty for the most relevant economic decisions lies in the properties of reality, namely the possibility of creativity and structural change in several aspects. Thus, in what refers to the roots of strong uncertainty, the difference between the various approaches does not seem to be very significant. The works that emphasize incomplete evidence (low weight) and unreliable knowledge but do not deal with the possibility of creativity and structural change could be seen as using a notion of uncertainty strong enough to go beyond standard EU theory. The emphasis on the possibility of creativity and structural change may be related to an even stronger notion of uncertainty, which I believe is the

most relevant for the economist. However, it is still to be better evaluated whether these approaches necessarily lead to divergent positions about the possible gradability of uncertainty. At any rate, I argue that uncertainty implies some lack of knowledge, but not complete ignorance, and therefore should be seen as gradable, in ordinal terms.

Endnotes

1. Faculty of Economics and Politics, University of Cambridge. I would like to thank, without implicating, Paul Davidson, Geoff Harcourt, Gay Meeks, Jochen Runde, Alessandro Vercelli and two anonymous referees, for comments on previous versions of this paper. Financial support from CNPq (Brazil) is also gratefully acknowledged.
2. Thus, strong uncertainty is not related to the context in which Herbert Simon's notion of bounded rationality is applied, which corresponds to what Dosi and Egidi (1991) call procedural uncertainty, nor is it related to Heiner's (1983) competence-difficulty (C-D) gap and Hayek's limits of reason (Vanberg, 1993, p.181).
3. On the strands that form Post Keynesianism, see e.g. Hamouda and Harcourt, 1988.
4. I am interested in the uncertainty involved in economic decisions. Thus, in principle my focus is on decision outcomes and consequently on events. However, the discussion of probabilities that refer to arguments can be adapted for my purposes, by discussing arguments about events. I have intentionally not mentioned states of the world (sometimes called states of nature), to which probability are

attached in EU theory, because there is a problem with the notion of state of the world if one wants to deal with uncertainty in a strong sense. I return to these points below.

5. In most of the *TP* Keynes tended to treat probable beliefs as arising from our apprehension of objective logical relations of partial implication between propositions. Later he accepted Ramsey's criticism of this. Runde (1994b) maintains that this issue does not affect the *TP* theory of comparative probability. According to Runde (1994b, pp.103, 108), this theory is compatible with different positions about the basis of our probable beliefs and requires simple judgements of preference, indifference, relevance and irrelevance. It should also be noted that one can reject Keynes's earlier commitment to objective logical probability relations while continuing to adhere to an epistemic conception of probability which is still different from the subjective probability theory.
6. Most authors take the first view. See, for example, Rutherford (1984, p.379n), Hoogduin (1987, p.57), Kregel (1987, pp.525-6), Carabelli (1988, pp.55-6) and Darity and Horn (1993, p.26). O'Donnell (1991c, pp.71-2) sees weight as 'one of the indefinables' of Keynes's system, but argues that weight 'reflects' or increases with the amount of relevant evidence. The second interpretation is favoured by Lawson (1987, p. 953n; 1988, p.49n) and, while recognising that Keynes has different concepts of weight in the *TP*, by Runde (1990) and McCann (1994, p.44). Runde convincingly argues that the second sense is the one required for connecting weight with confidence in *The General Theory*. Only in this sense may new evidence reduce weight and confidence (see also Cottrell, 1993, p.48).
7. In contrast, Winslow (1989, p.1179) defines fundamental uncertainty as 'the absence of a rational basis for ranking alternative courses of action'. He argues that expectations can be fundamentally uncertain without being uncertain in the *TP* sense either of non-numerical probabilities or low weight, but he does not clarify how. O'Donnell (1991a, pp.46-7, 54n) associates radical uncertainty with the *TP* notion of unknown probabilities (which are another possibility in the probability dimension, in addition to non-numerical and noncomparable probabilities). I have devoted a separate short paper to the relation between unknown probabilities and uncertainty in Keynes (Dequech, forthcoming). Here, it is enough to state my opinion that unknown probabilities, in the *TP* sense, are not suitable for a strong definition of uncertainty. Other notions of unknown probabilities might suit such a definition. For example, references to unknown probabilities are made in the literature on the Ellsberg paradox.
8. Elsewhere (Dequech, forthcoming), I argue against O'Donnell's (1991a, pp.30-1; also 1989, p.260 and McCann, 1994, pp.105-6) suggestion that we interpret this passage in terms of unknown probabilities.
9. See Lawson (1988, p.51) and O'Donnell (1991a, pp.57-8n; 1991b, pp.81-2). Davidson's view seems to have this characteristic in common with the frequency theory. However, this is not sufficient to identify him with that theory, as Feltz and Hoogduin (1988, p.109) do.

10. For example, the issue of structural change does not even appear in the analysis of the bets offered to subjects in Ellsberg's 1961 experiment and its replications and extensions (for a survey of these experiments, see Camerer and Weber, 1992, section 3.1). In these cases, reliability - and, in the *TP* framework, weight - is nevertheless involved. I return to this below.
11. Note that strict Bayesians may accept another distinction between risk and uncertainty, based on the existence or not of objective probabilities known to the decision maker (e.g., Harsanyi, 1977, p.87), while still claiming that they can deal with uncertainty.
12. In some presentations of EU theory, probabilities are attached directly to consequences. A list of consequences and their respective probabilities constitutes a prospect. The equivalence between these alternative ways of presenting EU theory is given by the fact that, as Vercelli (1996, p.5) notes, there is a 'one to one correspondence from states to consequences'.
13. This principle is the analogue in Savage's theory to the independence axiom in Von Neumann-Morgenstern's. It can be stated as follows: if the lottery P^* is preferred to the lottery P , then the mixture $\alpha P^* + (1-\alpha)P^{**}$ will be preferred to the mixture $\alpha P + (1-\alpha)P^{**}$ for all $\alpha > 0$ and P^{**} (Machina, 1987, p.127). Mixing each of two lotteries with a third one - in the same proportion in the two cases - does not change the ranking (see Anand, 1993, pp.13-4 for an example). Ellsberg examples show that the comparison between these alternatives depends also on the reliability of the available information. The preferences revealed in Ellsberg's experiments are inconsistent with the existence of additive probabilities, and in Savage's theory the sure thing principle is mainly responsible for the additivity of probabilities (Karni and Schmeidler, 1991, p.1803).
14. As in the *TP* case, uncertainty here refers to probability as a degree of belief. Again, a bridge between this and the analysis of economic decisions (whose outcomes depend on events) is established by discussing degrees of belief about the occurrence of events.
15. Risk would correspond to what Fishburn (1994, p.137) calls 'aleatory uncertainty' or 'chance that affects stochastic phenomena not yet consummated'. This he distinguishes from 'epistemic uncertainty' as 'uncertainty about the way things really are which is engendered by limited knowledge or lack of understanding'. The examples Fishburn uses to clarify these definitions indicate that, once 'epistemic uncertainty' is solved, one can calculate unambiguous probabilities and only 'aleatory uncertainty' is left. Epistemic uncertainty refers, for instance, to the fairness of a coin, to the specific contents of an urn with balls of different colours (as in the Ellsberg problem, which is Fishburn's concern), etc.
16. Likewise, Vercelli (1996, p.13) relates uncertainty aversion to weight and takes the degree of subadditivity as a measure of the degree of perceived uncertainty. See Runde (1995b, p.349n) for a criticism of the nonadditive prior approach.

17. As noted above, the definition of uncertainty as absence of a probability distribution does not make sense from the perspective of the subjective approach to probability. Realizing this, Vercelli (1991, pp.73-4) relates uncertainty to reliability. It is worth noting again that the distinction between probability and reliability can make sense in non-strict Bayesianism but not in strict versions.
18. Runde (1993, pp.385-6) states that Davidson's official epistemology requires a category like this, built along the lines of the *TP* or of Lawson (1987).
19. As Hoogduin (1987) also suggests, although apparently not noticing the difference between the two senses of weight.
20. When referring to his *TP* chapter on weight, Keynes himself uses the expression 'very uncertain' in the *GT* (p. 148n, emphasis added), which suggests that he treated uncertainty as gradable (also *CW* XIV, p.113).
21. Davidson, in contrast, denies it.
22. 'When is the weight of the argument sufficient to lead to a decision to act on its basis? To this question Keynes provides no answer, and suggests there is none' (Kregel, 1987, p. 526). For the sake of precision, I would say that there is no completely objective answer to this question. Feltz and Hoogduin (1988, pp.107-8) state that the lower the weight, the more scope for other influences than probabilities alone. In Gärdenfors and Sahlin's (1982, p.370) and in Vercelli's (1991, p.86) conception, discussed later, the desired or minimum reliability (which they associate with Keynes's weight) depends also on subjective variables. See also Bausor (1985, p.75).
23. For Runde (1990, pp.286-7) weight is not 'a psychological or subjective propensity'. Hoogduin (1987, p.59) considers it as an objective measure (also O'Donnell, 1989, p.72). Contrast with Kregel (1987, p.526).
24. It is not easy, though, to define knowledge, especially after the attacks on positivism in modern philosophy of science. The *TP* notion of knowledge, and particularly of direct knowledge, is surely not a perfect option. For example, it has some traces of empiricism. Lawson (1987, pp.957-61) deals with this and other problems and argues, to say the least, that Keynes is ambiguous in this regard.
25. Vercelli's (1991, pp.72-9) 'k-uncertainty' depends on the degree of reliability of probability distributions (being thus reminiscent of the multiple priors approach, discussed above). As in some generalizations of EU theory, uncertainty for him relates not to the absence of such a distribution but to its reliability. The case where no probability distribution is sufficiently reliable would be an extreme situation. Vercelli states that his notion of reliability of probability distributions is very similar to Keynes's weight (in the sense of the amount of evidence), the main difference being that Vercelli refers to probability of events rather than arguments. He does not consider this a very important difference (p. 78).
26. See also Darity and Horn (1993, p.30). Lawson (1987, p.963) suggests that it is possible to have probable (even if non-numerical) knowledge of people's conformity to a given convention. At any rate, Lawson (*idem, ibidem*) implies that

- probable knowledge is not the only possible type of knowledge under uncertainty, when he argues that 'knowledge can have a relative/absolute character (...) even in a non-probabilistic context'.
27. If I admit that some knowledge under uncertainty is possible and that uncertainty is gradable, in this sense I am referring to the degree of incompleteness of knowledge. Although this is similar to the complement of weight in the evidence's-degree-of-completeness sense, it does not necessarily have to be couched exactly in the terms of the *TP* theory of probability. At any rate, what I wrote above about weight not providing a totally precise measure of uncertainty applies also to the notion I propose here.
28. This can be used to criticize those who try to associate radical uncertainty with limited capabilities, for they cannot know if and when a superpowerful computer is going to be invented.
29. Shackle also makes this point (see Pheby, 1987, pp.24-5). This is also Popper's (1957) major argument for rejecting the belief that history has a predetermined end. Most certainly, there is in this issue an Austrian connection between Shackle and Popper, perhaps reinforced via Hayek, who was Shackle's supervisor at the London School of Economics (Harcourt, 1981, p.139), where Popper would work later. Indeed, in a note on the origins of his book, Popper mentions a seminar organized by Hayek at the LSE. Shackle was present at that talk (Popper, 1975, p.108).
30. See also Lawson (1985, p.921), Davidson (1991a, p.133) and Hamouda and Smithin (1988, pp.162-3).
31. In the *GT* (p. 252; see also p. 141), Keynes refers to 'epoch-making inventions' as a factor of instability for the prospective yield of capital-assets.
32. In a 1938 letter regarding Tinbergen's work, Keynes (*CW XIV*, p.287) again refers to the factors pointed out here: 'What place is allowed for non-numerical factors, such as inventions, politics, labour troubles, wars, earthquakes, financial crises?' See also another comment on Tinbergen (*CW XIV*, p.309).
33. 'The existence of a high degree of Keynesian uncertainty (...) causes the existence of exogenous expectations, which become another source of this kind of uncertainty themselves' (Feltz and Hoogduin, 1988, pp.115-6). See also Hoogduin and Snippe (1987, p.432n).
34. This principle is sometimes referred to as the principle of effective demand. Since the latter has, however, been given other meanings, including the one that associates it with the possibility of unemployment equilibrium, I adopt here a less ambiguous denomination. On the different meanings of the principle of effective demand, see O'Donnell (1989, p.236). It should be noted that I am confining the discussion to a monetary economy. One might try to argue that even in a non-monetary economy the future is uncertain, in which case uncertainty would not be related to the autonomy of spending money.
35. In a draft of *The General Theory*, Keynes (*CW XXIX*, pp.80-1) wrote: 'For the proposition that supply creates its own demand, I shall substitute the proposition

that expenditure creates its own income'. See also Shackle (1967, p.93) and Wells (1991, p.355). Kalecki's (1954, p.46) argument about the possibility of deciding one's expenditure but not one's revenue also underlies his implicit rejection of Say's Law.

References

- Anand P (1991) 'The Nature of Rational Choice and The Foundations of Statistics', *Oxford Economic Papers*, 43 (2), April.
- Anand P (1993) *Foundations of Rational Choice Under Risk*, Oxford: Clarendon.
- Arestis P (1992) *The Post-Keynesian Approach to Economics*, Aldershot: Edward Elgar .
- Bateman B (1987) 'Keynes's changing conception of probability', *Economics and Philosophy*, 3.
- Bateman B (1990) 'The Elusive Logical Relation: An Essay on Change and Continuity in Keynes's Thought', in Moggridge, D (ed) *Keynes, Macroeconomics and Method*, Perspectives on the History of Economic Thought, Vol. IV, Aldershot: Edward Elgar.
- Bateman B and Davis J (eds) (1991) *Keynes and Philosophy*, Aldershot: Edward Elgar.
- Bausor, R (1985) 'The Limits of Rationality', *Social Concept*, 2.
- Brown-Collier E (1985) 'Keynes's view of an organic universe: the implications', *Review of Social Economy*, 43 (1).
- Brown-Collier E and Bausor R (1988) 'The Epistemological Foundations of *The General Theory*', *Scottish Journal of Political Economy*, 35 (3).
- Camerer C and Weber M (1992) 'Recent Developments in Modelling Preferences: Uncertainty and Ambiguity', *Journal of Risk and Uncertainty*, 5
- Carabelli A (1988) *On Keynes's Method*, London: Macmillan.
- Carabelli A (1996) 'Speculation and reasonableness', presented at the Second Conference on Keynes, Knowledge and Uncertainty, Leeds, March.
- Carvalho F (1988) 'Keynes on probability, uncertainty, and decision making', *Journal of Post Keynesian Economics*, 11 (1).
- Cottrell A (1993) 'Keynes's theory of probability and its relevance to his economics', *Economics and Philosophy*, 9.
- Darity W and Horn B (1993) 'Rational expectations, rational belief, and Keynes's *General Theory*', *Research in the History of Economic Thought and Methodology*, 11.
- Davidson P (1978) *Money and the Real World*, London: Macmillan, 2e.
- Davidson P (1982) 'Rational expectations: a fallacious foundation for studying crucial decision-making processes', *Journal of Post Keynesian Economics*, 5 (2).
- Davidson P (1988) 'A technical definition of uncertainty and the long-run non-neutrality of money', *Cambridge Journal of Economics*, 12 (3).
- Davidson P (1989) 'The Economics of Ignorance or Ignorance of Economics',

- Critical Review*, 3 (3-4).
- Davidson, P (1991a) 'Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective', *Journal of Economic Perspectives*, 5 (1).
- Davidson P (1991b) *Controversies in Post Keynesian Economics*, Aldershot: Edward Elgar.
- Davidson P (1993) 'Austrians and Post Keynesians on economic reality: rejoinder to the critics', *Critical Review*, 7 (2-3).
- Davis J (1989) 'Keynes and organicism', *Journal of Post Keynesian Economics*, 12 (2).
- Davis J (1991) 'Review Essay: Keynes: Philosophy, Economics and Politics', *Review of Social Economy*, 49, (3).
- Davis J (1994a) 'On interpreting Keynes's philosophical thinking: Carabelli's On Keynes's Method and O'Donnell's Keynes: Philosophy, Economics and Politics: a review essay', *Research in the History of Economic Thought and Methodology*, 12.
- Davis J (1994b) 'Keynes's Philosophical Thinking', in Davis J (ed) *The State of Interpretation of Keynes*, Boston MA: Kluwer.
- Deprez J (1985) 'Time in a multi-industry, fixed-capital world', *Journal of Post Keynesian Economics*, 8 (2).
- Dequech D (1997a) 'On some arguments for the rationality of conventional behaviour under uncertainty', in Sardonì C (ed) *Themes in Political Economy*, London: Routledge.
- Dequech D (1997b) 'Expectations, confidence, liquidity preference and rationality under uncertainty', University of Cambridge, mimeo.
- Dequech D (forthcoming), 'A brief note on Keynes, unknown probabilities and uncertainty in a strong sense', *History of Economic Ideas*.
- Dosi G and Egidi M (1991) 'Substantive and procedural uncertainty - an exploration of economic behaviour in changing environments', *Journal of Evolutionary Economics*, 1 (2).
- Dow S (1995) 'Uncertainty about uncertainty', in Dow S and Hillard J (eds) *Keynes, Knowledge and Uncertainty*, Aldershot, UK: Edward Elgar.
- Dow A and Dow S (1985) 'Animal spirits and rationality', in Lawson T and Pesaran H (eds) *Keynes' Economics - Methodological Issues*, Armonk, NY: M E Sharpe.
- Dow S and Hillard J eds (1995) *Keynes, Knowledge and Uncertainty*, Aldershot: Edward Elgar.
- Dutt A K and Amadeo E (1990) *Keynes's Third Alternative? The Neo-Ricardian Keynesians and the Post Keynesians*, Aldershot: Edward Elgar.
- Ellsberg, D (1961) 'Risk, ambiguity and the Savage axioms', *Quarterly Journal of Economics*, 75.
- Feltz W and Hoogduin L (1988) 'Rational formation of expectations: Keynesian uncertainty and Davidson's (non)-ergodicity-criterium', *Metroeconomica*, 39 (2).
- Fishburn P (1994) 'A variational model of preference under uncertainty', *Journal of*

Risk and Uncertainty, 8.

- French S and Xie Z (1994) 'A Perspective on Recent Developments in Utility Theory', in Ríos S (ed) *Decision theory and decision analysis: trends and challenges*, Boston MA: Kluwer.
- Frisch D and Baron J (1988) 'Ambiguity and Rationality', *Journal of Behavioral Decision Making*, 1.
- Frowen S (ed) (1990) *Unknowledge and Choice in Economics*, London: Macmillan.
- Gärdenfors P and Sahlin N-E (1982) 'Unreliable Probabilities, Risk Taking, and Decision Making', *Synthèse*, 53, (3).
- Gerrard B (1992) 'From A Treatise on Probability to the General Theory: continuity or change in Keynes's thought?' in Gerrard B and Hillard J (eds) *The Economics of Rationality*, London: Routledge.
- Gerrard B (1995) 'Probability, uncertainty and behaviour: a Keynesian perspective', in Dow S and Hillard J (eds) *Keynes, Knowledge and Uncertainty*, Aldershot: Edward Elgar.
- Gerrard B ed (1992) *The Economics of Rationality*, London: Routledge.
- Gerrard B and Hillard J (eds) (1992) *The Philosophy and Economics of J M Keynes*, Aldershot: Edward Elgar.
- Gilboa I (1987) 'Expected utility with purely subjective non-additive probabilities' *Journal of Mathematical Economics*, 16.
- Gilboa I and Schmeidler D (1989) 'Maximin expected utility with a non-unique prior', *Journal of Mathematical Economics*, 18.
- Gillies D and Ietto-Gillies G (1991) 'Intersubjective probability and economics', *Review of Political Economy*, 3 (4).
- Hamouda O and Rowley R (1987) 'Troublesome probability and economics', *Journal of Post Keynesian Economics*, X (1).
- Hamouda O and Smithin J (1988) 'Some remarks on "Uncertainty and economic analysis"', *Economic Journal*, 98.
- Hamouda O and Harcourt G C (1988) 'Post-Keynesianism: from criticism to coherence?', *Bulletin of Economic Research*, 40, reprinted in Harcourt G C *On Political Economists and Modern Political Economy*, London:Routledge.
- Harcourt G C (1981) 'Notes on an economic querist: G L S Shackle', *Journal of Post Keynesian Economics*, 4 (1).
- Harcourt G C (1992) *On Political Economists and Modern Political Economy*, London: Routledge.
- Harsanyi J (1977) 'Advances in Understanding Rational Behavior', reprinted in Elster J (ed) (1986) *Rational Choice*, Oxford: Blackwell.
- Heiner R (1983) 'The origin of predictable behavior', *American Economic Review*, 73 (4).
- Hicks J (1977) *Economic Perspectives*, Oxford: Oxford UP.
- Hicks J (1979) *Causality in Economics*, New York: Basic Books.
- Hoogduin L (1987) 'On the difference between the Keynesian, Knightian and the "Classical" analysis of uncertainty and the

- development of a more general monetary theory', *De Economist*, 135 (1).
- Hoogduin L and Snippe J (1987) 'Uncertainty in/of macroeconomics: an essay on adequate abstraction', *De Economist*, 135 (4).
- Kalecki M (1954) *Theory of Economic Dynamics*, London: Allen & Unwin (2e 1965).
- Karni E and Schmeidler D (1991) 'Utility Theory with Uncertainty', in Hildenbrand W and Sonnenschein H eds *Handbook of Mathematical Economics*, vol. IV, Amsterdam: Elsevier.
- Kelsey D and Quiggin J (1992) 'Theories of choice under ignorance and uncertainty', *Journal of Economic Surveys*, 6 (2).
- Keynes J M (1936) *The General Theory of Employment, Interest and Money*, London: Macmillan.
- Keynes J M (1973a) *The Collected Writings of John Maynard Keynes*, vol. VIII, *A Treatise on Probability*, London: Macmillan/Royal Economic Society.
- Keynes J M (1973b) *The Collected Writings of John Maynard Keynes*, vol. XIV, London: Macmillan/Royal Economic Society.
- Keynes, J. M (1979) *The Collected Writings of John Maynard Keynes*, vol. XXIX, London: Macmillan/Royal Economic Society.
- Kregel J (1987) 'Rational Spirits and the Post Keynesian macrotheory of microfoundations', *De Economist*, 135 (4).
- Kregel J (1990) 'Imagination, Exchange and Business Enterprise in Smith and Shackle', in Frowen S (ed) *Unknowledge and Choice in Economics*, London: Macmillan.
- Kreps D (1990) *A Course in Microeconomic Theory*, New York: Harvester Wheatsheaf.
- Langlois R (1984) 'Internal organization in a dynamic context: some theoretical considerations', in Jussawalla M and Ebenfield H (eds) *Communication and Information Economics: New Perspectives*, Amsterdam: North-Holland.
- Langlois R (1986) 'Rationality, institutions, and explanation', in Langlois R (ed) *Economics as a process: essays in the new institutional economics*, Cambridge: Cambridge U P.
- Langlois R and Everett M (1992) 'Complexity, Genuine Uncertainty, and the Economics of Organization', *Human Systems Management*, 11.
- Lavoie M (1992) *Foundations of Post-Keynesian Economic Analysis*, Aldershot, UK: Edward Elgar.
- Lawson T (1985) 'Uncertainty and economic analysis', *Economic Journal*, 95.
- Lawson T (1987) 'The relative/absolute nature of knowledge and economic analysis', *Economic Journal*, 97.
- Lawson T (1988) 'Probability and uncertainty in economic analysis', *Journal of Post Keynesian Economics*, 11 (1).
- Machina M (1987) 'Choice Under Uncertainty: Problems Solved and Unsolved', *Journal of Economic Perspectives*, 1 (1).
- McCann C (1994) *Probability Foundations of*

Economic Theory, London: Routledge.

- Maki U, Gustafsson B and Knudsen C (eds) (1993) *Rationality, Institutions and Economic Methodology*, New York: Routledge.
- Meeks J G (1991) 'Keynes on the rationality of decision procedures under uncertainty: the investment decision', in Meeks, J G (ed) *Thoughtful Economic Man*, Cambridge: Cambridge U P.
- Moggridge D (ed) (1990) *Keynes, Macroeconomics and Method*, Perspectives on the History of Economic Thought, Vol IV, Aldershot, UK: Edward Elgar.
- O'Donnell R (1989) *Keynes: Philosophy, Economics and Politics*, London: Macmillan.
- O'Donnell R (1990a) 'Continuity in Keynes's conception of probability', in Moggridge D (ed) *Keynes, Macroeconomics and Method*, Perspectives on the History of Economic Thought, Vol IV, Aldershot, UK: Edward Elgar.
- O'Donnell R (1990b) 'An overview of probability, expectations, uncertainty and rationality in Keynes's conceptual framework', *Review of Political Economy*, 2 (3).
- O'Donnell R (1991a) 'Keynes on probability, expectations and uncertainty', in O'Donnell R (ed) (1991d) *Keynes as Philosopher-Economist*, New York: St. Martin's Press.
- O'Donnell R (1991b) 'Reply', in O'Donnell, R. ed (1991d) *Keynes as Philosopher-Economist*, New York: St. Martin's Press.
- O'Donnell R (1991c) 'Keynes's Weight of Argument and its Bearing on Rationality and Uncertainty', in Bateman B and Davis J (eds) (1991) *Keynes and Philosophy*, Aldershot: Edward Elgar.
- O'Donnell R (ed) (1991d) *Keynes as Philosopher-Economist*, New York: St. Martin's Press.
- Pheby J (1987) 'A New Perspective on Shackle's Keynesian Fundamentalism', *Journal of Economic Studies*, 14 (4).
- Popper K R (1957) *The Poverty of Historicism*, Boston: Beacon Press.
- Popper K R (1975) *Unended Quest*, London: Fontana.
- Rotheim R (1988) 'Keynes and the language of probability and uncertainty', *Journal of Post Keynesian Economics*, 11 (1).
- Rotheim R (1989-90) 'Organicism and the role of the individual in Keynes's thought', *Journal of Post Keynesian Economics*, 12 (2).
- Rotheim R (1995) 'Keynes on uncertainty and individual behaviour within a theory of effective demand', in Dow S and Hillard J (eds) *Keynes, Knowledge and Uncertainty*, Aldershot: Edward Elgar.
- Runde J (1990) 'Keynesian uncertainty and the weight of arguments', *Economics and Philosophy*, 6.
- Runde J (1991) 'Keynesian uncertainty and the instability of beliefs', *Review of Political Economy*, 3 (2).
- Runde J (1993) 'Paul Davidson and the Austrians: reply to Davidson', *Critical Review*, 7 (2-3).
- Runde J (1994a) 'Keynesian uncertainty and

- liquidity preference', *Cambridge Journal of Economics*, 18 (2).
- Runde J (1994b) 'Keynes After Ramsey: In Defence of A Treatise on Probability', *Studies in History and Philosophy of Science*, 25 (1).
- Runde J (1995a) 'Risk, uncertainty and Bayesian decision theory: a Keynesian view', in Dow S and Hillard J (eds) *Keynes, Knowledge and Uncertainty*, Aldershot: Edward Elgar.
- Runde J (1995b) 'Chances and Choices: Some Notes on Probability and Belief in Economic Theory', *The Monist*, 78 (3).
- Rutherford M (1984) 'Rational expectations and Keynesian uncertainty: a critique', *Journal of Post Keynesian Economics*, 6 (3).
- Sardoni C (ed) (1997), *Themes in Political Economy*, London: Routledge, forthcoming.
- Savage L (1954) *The Foundations of Statistics*, New York: Wiley.
- Schmeidler D (1989) 'Subjective probability and expected utility without additivity', *Econometrica*, 57.
- Schumpeter, J A (1943) *Capitalism, Socialism and Democracy*, New York: Harper (2e, 1950).
- Shackle G (1967) *The Years of High Theory*, Cambridge: Cambridge U P.
- Shackle G (1972) *Epistemics and Economics*, Cambridge: Cambridge U P.
- Shoemaker P (1982) 'The Expected Utility Model: Its Variants, Purposes, Evidence and Limitations', *Journal of Economic Literature*, 20 (2).
- Sugden R (1991) 'Rational Choice: A Survey of Contributions from Economics and Philosophy', *Economic Journal*, 101.
- Varian H (1992) *Microeconomic Analysis*, New York: Norton, 3e.
- Vanberg V (1993) 'Rational Choice, Rule-Following and Institutions: An Evolutionary Perspective', in Maki U, Gustafsson B and Knudsen C (eds) *Rationality, Institutions and Economic Methodology*, New York: Routledge.
- Vercelli A (1991) *Methodological foundations of macroeconomics: Keynes and Lucas*, Cambridge: Cambridge U P.
- Vercelli A (1995) 'From Soft Uncertainty to Hard Environmental Uncertainty', *Economie Appliquée*, 48 (2).
- Vercelli A (1996) 'Uncertainty, Rationality and Learning: a Keynesian perspective', presented at the Workshop on Realism and Economics, Cambridge, February.
- Wells P (1991) 'Keynes's *General Theory* Critique of the Neoclassical Theories of Employment and Aggregate Demand', *Review of Social Economy*, 49 (3).
- Winslow T (1989) 'Organic Interdependence, Uncertainty and Economic Analysis', *Economic Journal*, 99.
- Winslow T (1992) 'Keynes on rationality', in Gerrard B ed *The Economics of Rationality*, London: Routledge.