Economic Issues, Vol. 13, Part 1, 2008

Social Project Appraisal and Discounting for the Very Long Term

David Evans[1]

ABSTRACT

This paper considers the issue of discounting in relation to the appraisal of very long-term social projects. This is an increasingly important matter given the growing concerns over the environmental and safety impacts of many projects on future generations. Standard discounting practice trivialises the welfare impacts of projected costs and benefits on future populations, so a credible alternative procedure is required to avoid this 'write-off'. The British Treasury bases its published long-term discount factors on uncertainty concerning the value of the discount rate, although the factor values it has published seem inappropriate. The nature of this problem is examined and an alternative set of discount factors is presented for consideration. Additionally, a further set of factors is proposed for possible policy application in developed countries in cases where social project welfare impacts are not related to income. This new set is based only on uncertainty in the utility discount rate.

1. INTRODUCTION

THERE HAS RECENTLY BEEN a renewal of research interest in social discounting practices for the very long-term. This interest has resulted in the development of new theoretical approaches, following important earlier work in this area such as the Modified Discounting Method (Kula, 1988), and the Multi-Generational Value Model, (Bellinger, 1991). Much of the recent research has concentrated on providing theoretical justifications for time-declining discount rates and the work has been prompted by important policy concerns such as global warming, nuclear waste and, more generally, sustainable development. Important contributions to theory include the work of Chichilnisky (1997), Gollier (2002) and Weitzman (1998 and 2001).

The work of Weitzman (*ibid*) considers uncertainty in the discount rate: his approach is especially amenable to empirical investigation and has attracted considerable policy attention. For example, HM Treasury (2003) based its published schedule of declining discount rates for the UK on this particular approach. The Treasury followed the recommendations of OXERA (2002) but has applied the Weitzman approach inappropriately in its calculation of relevant discount factors. The construction of an appropriate series of time-declining discount rates and associated discount factors is an important matter since the standard application of the official British rate of 3.5 per cent trivialises the present value of projected cost and benefit impacts on future generations.

In the next section of the paper the essential theory is presented, together with an illustrative example. Also, the empirical work underpinning HM Treasury's schedule of time-declining discount rates is discussed briefly and the nature of the problem concerning the Treasury's published discount factors then explained. A comparison of the Treasury's figures with the discount factors that properly reflect the Weitzman approach reveals that the latter produce significantly higher present value welfare weights.

In section 3, a case is made for discounting only on the basis of the utility discount rate in the appraisal of projects where the benefits and costs mainly comprise environmental and health impacts. A plausible value range and different likelihood distributions for an uncertain utility discount rate are considered. The results show the persistence of non-trivial present value welfare weights far into the future, although the weights do exhibit some sensitivity to the assumed distribution of possible rates.

In the concluding section of the paper the main findings are summarised and a recommendation made that for many very long-term social projects discounting be based only on the utility discount rate. Alternative sets of discount factors are presented for consideration in relation to possible policy applications in different appraisal contexts.

2. The Weitzman approach and amendments to the Treasury's discount factors

Based on the work of Weitzman (1998, 2001), uncertainty concerning the value of the discount rate (r_i) produces a time-declining path for this variable based on the expected discount factor (d_t) associated with the distribution of possible rates at future time *t*. The equation for the discount factor is:

$$d_t = \sum L_i (1 + r_i)^{-t} \tag{1}$$

where, L_i = likelihood of rate r_i; t = number of years beyond the project implementation date; d_t = expected discount factor for future year *t*.

For each future year, the value of d_t can be converted into a certainty-equivalent discount rate (r_t*) using the equation below:

$$(1 + r_t*) = d_t^{-1/t} \qquad (2)$$

To demonstrate clearly that r_t* declines over time, in accordance with the theory embodied in equations (1) and (2), a simple example can be used in which it is assumed that r_i may take values in the 2-5 per cent range with equal likelihood. All the relevant figures are shown in Table 1, together with results for selected future years ranging from 30 to 400 years ahead. The example shows the falling values of r_t* over time, from 3.35 per cent for t =30 to just 2.42 per cent for t = 400.

Table 1: The time-declining discount rate: an illustration

	30 years	100 years	200 years	400 years
r (%)	$(1 + r)^{-30}$	$(1 + r)^{-100}$	$(1 + r)^{-200}$	$(1 + r)^{-400}$
2.0	0.5521	0.1380	0.0190	0.0004
2.5	0.4767	0.0846	0.0072	0.0001
3.0	0.4120	0.0520	0.0027	0.0000
3.5	0.3563	0.0321	0.0010	0.0000
4.0	0.3083	0.0198	0.0004	0.0000
4.5	0.2670	0.0123	0.0002	0.0000
5.0	0.2314	0.0076	0.0001	0.0000
d_t	0.3720	0.0495	0.0044	0.000071
$(1 + r_t*)$	1.0335	1.0305	1.0275	1.0242
r_t* (%)	3.3500	3.0500	2.7500	2.4200

1 r = discount rate.
2 d_t = Expected discount factor for future year t.
3 r_t* = Certainty-equivalent discount rate for future year t. See equation (2) for details of its calculation.

Drawing on the work of Newell and Pizer (2001), OXERA (2002) constructed a time-declining path for the British discount rate based on a starting figure of 3.5 per cent, the social time preference rate (*stpr*). Newell and Pizer (ibid) used a random walk model and past data on real long-term US government bond rates to simulate future bond rate paths and establish a probability distribution for future rates. See OXERA (2002, p35) for an illustration of the continuously declining discount rate based on the *stpr* and the underlying discount factors. For the convenience of practical policy application in cost-benefit analysis, a step schedule of rates approximating the continuously declining curve was introduced. HM Treasury (2003) uses these step rates in its appraisal guidance for government departments and the relevant figures are shown in Table 6.1 of the Green Book (HM Treasury, p99). In this paper, it is argued that the Treasury's published long-term discount factors fail to reflect the Weitzman approach as applied to the approximating step rates. The nature of the problem and its importance are considered below and calculations of the correct discount factors are explained and presented.

Revisions to the Treasury's long-term discount factors

HM Treasury (2003) does not provide any details of the calculations required to produce its table of long-term discount factors (ibid, p100). The intention, however, was to follow the procedure outlined in OXERA (2002) based on the Weitzman (1998 and 2001) approach. For the step schedule of discount rates, the value of the discount factor must increase locally at each change in the value of the step rate, before falling again. This is the small price paid for using a convenient, practical procedure designed to provide a decent approximation to the discount factors associated with the continuously time-declining discount rate. So, the step schedule discount factors are clearly given by the following equation:

$$d_t = (1+r_t)^{-t} \tag{3}$$

where r_t = step discount rate at time t.

The Treasury's discount factors (T), however, are based on the following equation that provides figures calculated from the weighted geometric mean of the step discount rates up to the marginal future year (t):

$$T = [\{(1+r_1)^{t1}(1+r_2)^{t2-30}(1+r_3)^{t3-75}(1+r_4)^{t4-125}(1+r_5)^{t5-200}(1+r_6)^{t6-300}\}^{1/t}]^{-t} \tag{4}$$

where,

$0 < t1 < 31$; $30 < t2 < 76$; $75 < t3 < 126$; $125 < t4 < 201$; $200 < t5 < 301$; $t6 > 300$

$$t = t1 + (t2 - 30) + (t3 - 75) + (t4 - 125) + (t5 - 200) + (t6 - 300)$$

For $t < 31$ set $t = t1$; for $30 < t < 76$ set $t = t2$; for $75 < t < 126$ set $t = t3$; for $125 < t < 201$ set $t = t4$; for $200 < t < 301$ set $t = t5$; for $t > 300$ set $t = t6$.

The time-weighted geometric mean of the relevant step rates (plus 1) is given by the entire expression contained within square brackets in equation (4). The relevant step rates based on the social time preference rate (*stpr*) are as follows:

$$r_1 = 0.035;\ r_2 = 0.03;\ r_3 = 0.025;\ r_4 = 0.02;\ r_5 = 0.015;\ r_6 = 0.01$$

The weighted geometric mean rate is clearly higher than the marginal step discount rate (r_t) in equation (3), for all time horizons beyond 30 years. Therefore, the Treasury's long-term discount factors (*T*) are too low compared with the correct step rate discount factors (d_t) given by equation (3). A comparison of discount factors using these different procedures is shown in Table 2. The term d_t is re-labelled '*W*' in this table in order to highlight the fact that it is consistent with a step rate approximation to the Weitzman approach.

Table 2: Discount factors using different discounting procedures

Year (*t*)	Standard rate (*S*) (3.5%)	Treasury step rate (*T*)	Weitzman step rate (*W*)
30	0.3563	0.3563	0.3563
50	0.1790	0.1973	0.2281
75	0.0758	0.0942	0.1089
100	0.0321	0.0508	0.0846
125	0.0136	0.0274	0.0457
150	0.0057	0.0167	0.0513
200	0.0010	0.0062	0.0190
250	0.0002	0.0029	0.0242
300	0.0000	0.0014	0.0115
400	0.0000	0.0005	0.0187
500	0.0000	0.0002	0.0069

Notes:

1 The relevant step discount rates are as follows:

0 – 30 yrs	3.5%	126 – 200 yrs	2.0%
31 – 75 yrs	3.0%	201 – 300 yrs	1.5%
76 – 125 yrs	2.5%	301 + yrs	1.0%

2 For the calculation of *T* equation (4) is used and for *W* equation (3) applies.

3 Examples of calculations for *S*, *T* and *W* at $t = 100$:

a. $S = 1.035^{-100} = 0.0321$

b. $T = 1.035^{-30} \times 1.030^{-45} \times 1.025^{-25} = 0.0508$

c. $W = 1.025^{-100} = 0.0846$

HM Treasury should be using '*W*' rather than '*T*' as its long-term discount factors and its publication of the latter in the 2003 Green book (HM Treasury, 2003, p100) may well involve government funding decisions that are, as a result, likely to under-allocate resources to future generations. Table 2 also includes the discount factors (*S*) based on standard discounting at 3.5 per cent. Inspection of the 3 sets of discount factors reveals that only the correctly applied Weitzman approach (*W*) produces any obvious present value weighting benefits for future generations, compared with standard discounting practice (*S*).

3 Discounting based only on the utility discount rate

In Table 2, the calculated discount factors are based on a starting discount rate of 3.5 per cent (the Treasury's preferred official rate for the UK based on social time preference). This particular rate is based on an assessment of the evidence concerning the values of the component parameters of the *stpr* provided in surveys of empirical work (see, especially, Cowell and Gardiner, 1999, Pearce and Ulph, 1999, and OXERA, 2002). The equation for the *stpr* is:

$$stpr = p + eg \tag{5}$$

where p = utility discount rate; e = elasticity of marginal utility of consumption; g = projected average annual rate of long-term growth of real per capita consumption.

Based on the aforementioned empirical evidence, HM Treasury selects p = 1.5 per cent, e = 1.0 and g = 2.0 per cent. However, in relation to many very long-term social projects and policies, it can be argued that the term '*eg*' in equation (5) is not relevant and should be dropped leaving only the utility discount rate (p). As suggested by Spackman (2004, section vii, pp 499-500):

> Many of the very long-term impacts of policy interest are direct impacts on welfare in terms such as safety or health, whose marginal utility may be almost independent of income. In such cases, it is this pure time preference that determines the extent to which they are discounted over time.

So, for many very long-term project appraisals there is a good case for focusing only on the utility discount rate and thus setting $stpr = p$. The appropriate value of p is subject to a great deal of controversy, with widely differing views expressed concerning what should be the preferred figure. Ramsey (1928), Pigou (1932), Price (1989) and Broome (1992) all suggested a zero rate, while Little and Mirrlees (1974), Stern (1977) and Kula (1985) suggested an appropriate rate of 2 per cent or more. Scott (1989) and Newbery (1992) consider that rates close to 1 per cent are appropriate, while Pearce and Ulph (1999) provide a 'best estimate' of almost 1.5 per cent. This last estimate is consistent with the UK Treasury's preferred *p* value of 1.5 per cent as expressed in its latest appraisal guidance (HM Treasury, 2003), although in its previous guidance the lower value of 1 per cent was assumed, HM Treasury (1997). Given these wide-ranging views on the relevant rate, there could be a case for assuming a possible value range of 0-2 per cent.

Uncertainty-based time-declining discount rates can be constructed for the preferred *p*-value range on the basis of different assumptions concerning the distribution of possible rates. For equal likelihood of *p*-values, the expected discount factors and certainty-equivalent discount rates are shown in Table 3. An alternative set of results, based on normally-distributed *p*-values, is presented in Table 4. In this latter case, the present value weights (discount factors) are significantly lower, indicating that the assumption made about the distribution of rates is important.

Table 3 Discount factors (DF) based on *r* = *p*: uniformly distributed values

r (%)	DF 30 yrs	DF 100 yrs	DF 200 yrs	DF 400 yrs
0.1	0.9705	0.9049	0.8188	0.6704
0.3	0.9140	0.7411	0.5493	0.3017
0.5	0.8610	0.6073	0.3688	0.1360
0.7	0.8112	0.4978	0.2478	0.0614
0.9	0.7643	0.4082	0.1666	0.0278
1.1	0.7202	0.3349	0.1121	0.0126
1.3	0.6788	0.2748	0.0755	0.0057
1.5	0.6398	0.2256	0.0509	0.0026
1.7	0.6030	0.1853	0.0343	0.0012
1.9	0.5686	0.1523	0.0232	0.0005
d_t (0% - 2%)	0.7531	0.4332	0.2447	0.1220
(r_t*)	(0.95%)	(0.84%)	(0.71%)	(0.53%)

Notes:

For definitions of *r*, d_t and r_t* see notes to Table 1.

The controversy concerning the precise nature of *p* and the wide-ranging views concerning an appropriate value for the UK, or indeed for any other developed country, are factors supporting the assumption of equal likelihood of *p*-values within the broad value range of 0-2 per cent. It thus follows that the results shown in Table 3 are worthy of special attention from a policy application perspective. So, for a 0-2 per cent range for r (where $r = p$), the table reveals that the mean discount factor (d_t) drops from 75 per cent at 30 years to 12 per cent after 400 years. So, projected benefits and costs for distant future populations still have a non-trivial welfare weighting in present value terms.

Table 4: Discount factors (DF) based on $r = p$: normally distributed values

Range of r values (%)	Likelihood (l)	r %	DF 30 yrs	DF 100 yrs	DF 200 yrs	DF 400 yrs
0.0 - 0.2	0.0082	0.1	0.0080	0.0074	0.0067	0.0055
0.2 - 0.4	0.0277	0.3	0.0253	0.0205	0.0152	0.0084
0.4 - 0.6	0.0792	0.5	0.0682	0.0481	0.0292	0.0108
0.6 - 0.8	0.1592	0.7	0.1291	0.0792	0.0394	0.0098
0.8 - 1.0	0.2257	0.9	0.1725	0.0921	0.0376	0.0063
1.0 - 1.2	0.2257	1.1	0.1625	0.0756	0.0253	0.0028
1.2 - 1.4	0.1592	1.3	0.1081	0.0437	0.0120	0.0009
1.4 - 1.6	0.0792	1.5	0.0507	0.0179	0.0040	0.0002
1.6 - 1.8	0.0277	1.7	0.0167	0.0051	0.0009	0.0000
1.8 - 2.0	0.0082	1.9	0.0047	0.0012	0.0002	0.0000
d_t			0.7458	0.3908	0.1705	0.0447
(r_t*)			(0.98%)	(0.94%)	(0.89%)	(0.78%)

Notes

1 Based on the properties of the normal distribution and the assumed value range for r, $sr = 0.3333 = 1/6$ (2); mean $r = 1.0\%$.

2 In the main table, all discount factors (DF) have been weighted by the relevant likelihoods (l) of r based on the normal distribution.

3 For definitions of r, d_t and r_t* see notes to table 1.

4. Concluding comments

HM Treasury's published discount factors for the very long-term (HM Treasury, 2003, p100) are too low and do not reflect the appropriate calculations based on discounting under uncertainty following the Weitzman approach (Weitzman, 1998 and 2001). Instead of being based on the relevant marginal step discount rates, they are calculated from the time-weighted geometric mean of the relevant step rates up to year t for $t = 1$ to 500. The correctly calculated discount factors (W) are shown alongside HM Treasury's figures (T), in Table 2, for selected years. The correct figures are noticeably higher, meaning that the application of the Treasury's published discount factors in social project appraisal must result in an unintended bias working against the interests of future generations. The obvious implication is for HM Treasury to revise its published discount factors at the earliest opportunity.

For many very long-term projects, the most important net benefits consist of environmental and safety welfare impacts that are not related to income and should therefore be discounted at the lower utility discount rate ($r = p$). Based on both the views held by economists and empirical evidence, a plausible value range for r is 0-2 per cent. Uncertainty concerning the value of r produces the discount factors shown in Tables 3 and 4. The former are based on an equal likelihood of possible values while those presented in Table 4 reflect a normal distribution. In this paper, the preferred results are the higher discount factors associated with the equal likelihood case. It is for policy-makers to decide the most appropriate case, but the publication of a set of discount factors based only on the utility discount rate is recommended as an additional item for inclusion in HM Treasury's next official appraisal guidance.

In relation to the utility discount rate, the plausible value range of 0-2 per cent may well apply to all developed countries. If so, then the results in either Table 3 or 4 would yield a common set of discount factors for application in very long-term social project appraisals involving cost and benefit impacts that, for the most part, are not income-related. This would help, for example, in the co-ordination of international policy with respect to matters of global concern, such as investment in energy and the mitigation of the adverse impacts of climate change. The relevant discount factors should be reviewed periodically, say every five years, in the light of new evidence regarding appropriate value ranges for all component parameters of the *stpr*. This evidence may involve new methodologies and the application of new statistical techniques, in addition to simple updating of empirical work based on established methods.

Accepted for publication: 25 May 2007

Endnotes

1. Oxford Brookes University, Oxford OX33 1HX. djevans@brookes.ac.uk

References

Bellinger W K (1991) 'Multigenerational value: modifying the modified discounting method', *Project Appraisal*, 6, 101-108.

Broome J (1992) *Counting the Cost of Global Warming*, Cambridge: White Horse Press.

Chichilnisky G (1997) 'What is sustainable development?', *Land Economics*, 73, 467-491.

Cowell F A and Gardiner K (1999) Welfare weights (STICERD), London School of Economics, Economics Research Paper 20.

Gollier C (2002) 'Discounting an uncertain future', *Journal of Public Economics*, 85, 149-166.

HM Treasury (1997) *Appraisal and Evaluation in Central Government* (The Green Book), London: HMSO.

HM Treasury (2003) *Appraisal and Evaluation in Central Government* (The Green Book), London: HMSO.

Kula E (1985) 'An empirical investigation on the social time preference rate for the UK', *Environment and Planning*, 17, 199-217.

Kula E (1988) 'Future generations: the modified discount factor', *Project Appraisal*, 3, 85-88.

Little I M D and Mirrlees J A (1974) *Project Appraisal and Planning for Developing Countries*, London: Heinemann Educational Books Ltd.

Newbery D (1992) 'Long-term discount rates for the Forest Enterprise', Department of Applied Economics, Cambridge University, for the UK Forestry Commission, Edinburgh.

Newell R and Pizer W (2001) 'Discounting the benefits of climate change mitigation: how much do uncertain rates increase valuations?', Economics Technical Series, Arlington, VA: Pew Center on Global Climate Change.

OXERA (2002) *A Social Time Preference Rate for Use in Long-term Discounting: A Report for the Office of the Deputy Prime Minister* (ODPM), The Department for Transport (DfT) and The Department for Environment, Food and Rural Affairs (DEFRA), London.

Pearce D and Ulph D (1999) 'A social discount rate for the UK', in Pearce D W (ed) *Economics and the Environment: Essays on Ecological Economics and Sustainable Development*, Cheltenham: Edward Elgar.

Pigou A C (1932) *The Economics of Welfare*, London: Macmillan.

Price C (1989) *The Theory and Applications of Forest Economics*, Oxford: Basil Blackwell.

Ramsey F P (1928) 'A mathematical theory of saving', *Economic Journal*, 38, 543-559.

Scott M F G (1989) *A New View of Economic Growth*, Oxford: Clarendon Press.

Spackman M (2004) 'Time discounting and of the cost of capital in government', *Fiscal Studies*, 25, 467-518.

Stern H N (1977) 'Welfare weights and the elasticity of marginal utility of income', in Artis M and Norbay R (eds) *Proceedings of the Annual Conference of the Association of University Teachers of Economics*, Oxford: Blackwell.

Weitzman M (1998) 'Why the far distant future should be discounted at its lowest possible rate', *Journal of Environmental Economics and Management*, 36, 201-208.

Weitzman M (2001) 'Gamma discounting', *American Economic Review*, 91, 261-271.