

Optimal Monetary Policy under Risk *and* Uncertainty

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ABSTRACT

This paper seeks to characterise optimal monetary policy rules in the presence of risk and uncertainty. I explore a situation in which the true parameters and the true structure of the economy are unknown to the policymaker, and he is reluctant to make a decision based on a single distribution estimate (i.e. he faces Knightian uncertainty). I show analytically that if the policymaker does not know the true structure of the economy he will be more cautious than in the case of only parameter risk. Further, I show that Knightian uncertainty can also lead to an extra precautionary motive when one considers its interaction with parameter risk. In a simple exercise, I provide empirical estimates that demonstrate that adjustments due to parameter and structural risk and Knightian uncertainty can potentially be quite large.

The Federal Reserve's experiences over the past two decades make it clear that uncertainty is not just a pervasive feature of the monetary policy landscape; it is the defining characteristic of that landscape. The term 'uncertainty' is meant here to encompass both 'Knightian uncertainty', in which the probability distribution of outcomes is unknown, and 'risk', in which uncertainty of outcomes is delimited by a known distribution. In practice, one is never quite sure what type of uncertainty one is dealing with in real time, and it may be best to think of a continuum ranging from well-defined risks to the truly unknown. — Alan Greenspan (2004)

1. INTRODUCTION

HOW SHOULD MONETARY POLICY be selected in the face of uncertainty? This long-debated question has recently been the theme of a large body of research. In essence, defining what the optimal policy should be reduces to identifying the 'type' of uncertainty that the policymaker faces.² It is well known, for example, that when there is only additive risk and preferences are linear-quadratic the policymaker should act as if the expected values were the true values.

Brainard (1967) shows, however, that under parameter (multiplicative) risk the policy response is more cautious than in the case of certainty. Brainard's 'conservatism principle' is an intuitive candidate to reconcile the

gradualism commonly observed in monetary policy movements and the much more aggressive behaviour often implied by optimising models under certainty equivalence. Most empirical studies, however, find that the attenuation effect of parameter uncertainty is not nearly enough to explain the data (e.g. Rudebusch, 2001).

In contrast to the standard Bayesian approach that seeks to characterise policy in terms of risks, recent research (e.g. Hansen and Sargent 2003; Giannoni, 2002, 2006) has also focused on policymaking in the face of Knightian uncertainty. This line of research has found that, in general, the optimal policy is even more aggressive to changes in the economy than the certainty-equivalent case.

Although the literature has come a long way in assessing policymaking in the face of risks or uncertainties, it is clear from the preceding remarks of the former Fed Chairman that *both* are important determinants of monetary policy in the US and around the world. Therefore, this paper seeks to characterise optimal monetary policy rules in the presence of risk *and* uncertainty. Specifically, I analyse a situation in which the true parameters and the true structure of the economy are unknown to the policymaker, and he is reluctant to make a decision based on a single estimate of the parameters' distribution. In doing so, I explore whether the model can explain the monetary policy gradualism usually observed in the data.

Following Cogley and Sargent (2005) and Brock, Durlauf, and West (2003, 2004) I assume that the monetary authority applies a technique known as Bayesian Model Averaging to optimally guard for structural risk.³ Using this approach to derive an optimal monetary rule I show that, in addition to 'Brainard's conservatism principle' due to parameter risk, structural risk unambiguously leads to a more cautious policy response than the *average* model considered.

Next, to describe the preferences of the monetary authority under Knightian uncertainty I use the so called *e*-contamination model presented by Epstein and Wang (1994), where the decision-maker has a reference probability distribution (of the parameters vector) but does not trust this specification completely. As a result of this uncertainty, the optimal instrument is one that minimises a standard Bayesian risk function (accounting for structural risk as described above) but gives a larger weight to the worst-case scenario.

Using a numerical simulation, I show that the conventional wisdom that Knightian uncertainty leads to a stronger policy response is largely dependent on its interaction with parameter risk (in the sense of the coefficients' variance). Further, in a simple exercise, I show that when one accounts for all 'types' of uncertainty, the precautionary motive can be quite large and the implied optimal rule is close to the celebrated Taylor rule.

In the next section I present the model(s) of the economy. In section 3 I derive the optimal policy rule for a standard Bayesian regulator who faces parameter and structural risks. In section 4 I depict the optimal policy in the

presence of Knightian uncertainty. In section 5 I present empirical estimates while section 6 concludes.

2. THE STRUCTURE OF THE ECONOMY

I depict the structure of the economy by means of a very simple model of output and inflation presented by Svensson (1997) and extended to include parameter risk by Estrella and Mishkin (1999). In particular, the model consists of three equations: a dynamic Phillips curve where inflation ρ depends on its lag, on a lag of the output gap y , and on a vector of other predetermined variables x ; an IS curve where the output gap depends on its lag, the monetary policy instrument r (say the Fed funds rate), and the mentioned exogenous variables; finally, the third equation defines the dynamics of the exogenous variables. In contrast to Svensson's analysis I consider the case where neither the parameters nor the structure of the economy are known with certainty. I model structural risks by assuming that the central bank considers K models that include all possible combinations of j exogenous variables (i.e. $K=2^j$). Formally, model i ($i=1,2,\dots,K$) is given by

$$\pi_{t+1,i} = \pi_t + a_{1,i}y_t + \sum_{j=3}^n a_{j,i}x_{t,i,j} + \varepsilon_{t+1,i} \quad (1)$$

$$y_{t+1,i} = b_{1,i}y_t - b_{2,i}r_t + \sum_{j=3}^n b_{j,i}x_{t,i,j} + v_{t+1,i} \quad (2)$$

$$x_{t+1,i,j} = c_{j,i}x_{t,i,j} + \mu_{t+1,i,j} \quad (3)$$

where the subscript i indicates that the variable or parameter correspond exclusively to this particular model.

Since the instrument affects inflation with a lag of two periods it is convenient to write the reduced-form expression (for inflation two periods ahead) in terms of current values of the variables and period $t+1$ and $t+2$ disturbances

$$\pi_{t+2,i} = \pi_t + \kappa_{1,i}y_t - \kappa_{2,i}r_t + \sum_{j=3}^n \kappa_{j,i}x_{t,i,j} + \zeta_{t+2,i} \quad (4)$$

where I have defined

$$\kappa_{1,i} \equiv a_{1,i}(1+b_{1,i}), \quad \kappa_{2,i} \equiv a_{1,i}b_{2,i}, \quad \text{and} \quad \kappa_{j,i} \equiv a_{j,i}(1+c_{j,i}) + a_{1,i}b_{j,i}$$

$$\zeta_{t+2,i} \equiv a_{1,i}v_{t+1,i} + a_{2,i} \sum_{j=3}^n \mu_{t+1,i,j} + \varepsilon_{t+1,i} + \varepsilon_{t+2,i}$$

Note that, despite its simplicity, the model space can potentially accommodate an enormous amount of information in the vector of exogenous variables. The motivation of such data-rich environment is twofold. First, recent research (e.g. Bernanke and Boivin, 2003; Stock and Watson, 2003) has shown that the use of a wider information set might improve inflation's forecast accuracy. Second, and probably related to the former, in practice central bankers use a very large number of variables to set policy.

3. OPTIMAL POLICY UNDER PARAMETER AND STRUCTURAL RISKS

The problem of a Bayesian regulator who faces parameter and structural risks can be characterised as follows

$$\text{Min}_{\theta} \int l(p, \theta) \Pr(\theta|D) d\theta \tag{5}$$

where $l(p, \theta)$ represents a loss function yet to be defined, that depends on p , the policy instrument that has support P , and θ (with support Θ) which represents the quantities that affect the function. $\Pr(\theta|D) = \sum_{i=1}^K \Pr(\theta|D, i) \Pr(i|D)$

represents the average of the posterior distributions of θ (conditional in the data D and the particular model i), weighted by the posterior model probabilities $\Pr(i|D)$. (This is known as Bayesian Model Averaging⁴).

In the present context, we will need an estimate of the mean and variance of θ . Leamer (1978) shows that, after accounting for structural risk, these estimates are

$$E(\theta|D) = \sum_{i=1}^K \Pr(i|D) E(\theta|D, i) \tag{6}$$

and

$$\text{Var}(\theta|D) = \sum_{i=1}^K \Pr(i|D) \text{Var}(\theta|D, i) + \sum_{i=1}^K \Pr(i|D) [E(\theta|D, i) - E(\theta|D)]^2 \tag{7}$$

To relate (5) to the problem at hand, suppose that the policymaker sets the instrument in period t to minimise the squared deviation of the current inflation forecast (for each model) from the inflation target⁵. In particular, the policy objective is to minimise, for each model i

$$E_t(\pi_{t+2, i} - \pi^*)^2 = (E_t \pi_{t+2, i} - \pi^*)^2 + \text{Var}(\pi_{t+2, i}) \tag{8}$$

Assuming that the central bank does not know the coefficients of the equations and that these coefficients are orthogonal across equations and the disturbances, the conditional mean and variance of inflation for model i are given by

$$E_t \pi_{t+2,i} = \pi_t + \kappa_{1,i} y_t - \kappa_{2,i} r_t + \sum_{j=3}^n \kappa_{j,i} x_{t,i,j} \quad (9)$$

and

$$Var(\pi_{t+2,i}) = \left[\begin{array}{l} \sigma_{\kappa_{1,i}}^2 y_t^2 + \sigma_{\kappa_{2,i}}^2 r_t^2 + \sum_{j=3}^n \sigma_{\kappa_{j,i}}^2 x_{t,i,j}^2 + (\omega_{1,i} y_t + \omega_{2,i,j} x_{t,i,j}) r_t \\ + \sigma_{\zeta,i}^2 + \omega_{1,i,j} y_t x_{t,i,j} \end{array} \right] \quad (10)$$

where

$$\begin{aligned} \sigma_{\kappa_{1,i}}^2 &\equiv a_{1,i}^2 \sigma_{b_{1,i}}^2 + \sigma_{a_{1,i}}^2 (1 + b_{1,i})^2 + \sigma_{b_{1,i}}^2 \sigma_{a_{1,i}}^2, & \sigma_{\kappa_{2,i}}^2 &\equiv a_{1,i}^2 \sigma_{b_{2,i}}^2 + \sigma_{a_{1,i}}^2 b_{2,i}^2 + \sigma_{a_{1,i}}^2 \sigma_{b_{2,i}}^2, \\ \sigma_{\kappa_{3,i}}^2 &\equiv a_{2,i}^2 \sigma_{c,i}^2 + \sigma_{a_{2,i}}^2 (1 + c_i)^2 + \sigma_{a_{2,i}}^2 \sigma_{c,i}^2 + a_{1,i}^2 \sigma_{b_{3,i}}^2 + \sigma_{a_{1,i}}^2 b_{3,i}^2 + \sigma_{b_{3,i}}^2 \sigma_{a_{1,i}}^2 \\ \omega_{1,i} &\equiv 2 \left(\sigma_{a_{1,i}}^2 (1 + b_{1,i}) b_{2,i} - \sigma_{(b_1, b_2), i} \right) & \omega_{2,i,j} &\equiv 2 \left(\sigma_{a_{1,i}}^2 b_{2,i} \sum_{j=3}^n b_j - \sum_{j=3}^n \sigma_{(b_2, b_j), i} \right) \\ \omega_{1,i,j} &\equiv 2 \left(\sigma_{a_{1,i}}^2 (1 + b_{1,i}) \sum_{j=3}^n b_j - \sum_{j=3}^n \sigma_{(a_1, a_j), i} - \sum_{j=3}^n \sigma_{(b_1, b_j), i} \right) \end{aligned}$$

Writing (9) and (10) in more compact form, $E_t \pi_{t+2,i} = A_i' Z_{t,i}$ and $Var(\pi_{t+2,i}) = W_{t,i}' B_i W_{t,i}$. I can now rewrite (5) as

$$Min_{\pi^*} \left[\begin{array}{l} \sum_{i=1}^K \Pr(i|D) (A_i' Z_{t,i} - \pi^*)^2 + \sum_{i=1}^K \Pr(i|D) W_{t,i}' B_i W_{t,i} \\ + \sum_{i=1}^K \Pr(i|D) \left[A_i' Z_{t,i} - \sum_{i=1}^K \Pr(i|D) A_i' Z_{t,i} \right]^2 \end{array} \right] \quad (11)$$

Using (11) it is possible to find a policy rule that accounts for both, parameter and structural risk. First, however, I present the certainty equivalent rule as a benchmark.

3.1. Certainty equivalent rule

To solve for the certainty equivalent optimal rule, suppose that the policymaker knows (or believes) that model i is the true model of the economy. Then, in the case in which there is only additive risk, the variance of inflation reduces to

$$Var(\pi_{t+2,i}) = \sigma_{\epsilon,i}^2$$

while the mean is given by (9). Thus, the optimal policy rule is given by

$$r_t = \frac{(\pi_t - \pi^*) + \kappa_{1,i} y_t + \sum_{j=3}^n \kappa_{j,i} x_{t,i,j}}{\kappa_{2,i}} \tag{12}$$

This feedback rule has the same form as the one proposed by Taylor (1993). Here, however, the response coefficients might be different from the ones suggested in the aforementioned paper and the monetary authority responds to other variables in addition to the output gap and the inflation gap.

3.2. Solution under parameter and structural risks

When the policymaker does not know with precision either the true parameters in (1)-(3) or the true structure of the economy it is simple to verify that the optimal interest rate rule takes the form

$$r_t^0 = \frac{\sum_{i=1}^K \Pr(i|D) \left[\kappa_{2,i} (\pi_t - \pi^*) + \left(\kappa_{1,i} \kappa_{2,i} - \frac{\omega_{1,i}}{2} \right) y_t + \sum_{j=3}^n \left(\kappa_{2,i} \kappa_{j,i} - \frac{\omega_{2,i,j}}{2} \right) x_{t,i,j} \right]}{\sum_{i=1}^K \Pr(i|D) \left(\kappa_{2,i}^2 + \sigma_{\kappa_{2,i}}^2 + (\kappa_2^A - \kappa_{2,i})^2 \right)} \tag{13}$$

where $\kappa_2^A \equiv \sum_{i=1}^K \Pr(i|D) \kappa_{2,i}$ represents the expected value of the parameter after averaging across the models. Intuitively, the optimal policy rule is a weighted average of the optimal rules for each model under consideration, with the weights given by the posterior model probabilities.

By comparing (12) to (13) we see that, due to the variance of the policy instrument's parameter $\sigma_{\kappa_{2,i}}^2$, the policy response is more cautious than the certainty equivalence rule for the average model, which is Brainard's famous result.⁶ The term $(\kappa_2^A - \kappa_{2,i})^2$ is due to structural risk. Since this term is strict ly positive we have the following

Proposition: *Suppose the policymaker does not know the true structural model of the economy. Then, this structural risk leads him to set a policy response that is more cautious.*

The intuition of this result was provided by Brock *et al* (2003 p.12) in a general framework of policy analysis under structural risk and is a straightforward extension of Brainard's conservatism principle. Since the policymaker cannot tell which of the models is the true one, the posterior variance in (7) is larger than in the case of only parameter risk due to the variance across models of the expected value of the response coefficients, i.e. because

$$\sum_{i=1}^K \Pr(i|D) [E(\theta|D,i) - E(\theta|D)]^2 = \sum_{i=1}^K \Pr(i|D) (\kappa_2^A - \kappa_{2,i})^2 > 0$$

This additional risk makes him respond less aggressively to changes in the inflation rate, the output gap, and the exogenous variables. Whether this adjustment is economically significant is an empirical issue that I address in a later section.

4. SEARCHING FOR ROBUSTNESS WITH UNCERTAIN DISTRIBUTIONS

To incorporate robustness under Knightian uncertainty I adapt the preferences described in Epstein and Wang (1994).⁷ In particular, I assume that the policymaker's objective is to minimise the following function⁸

$$(1-e) \int_{\Theta} l(p, \theta) \Pr(\theta|D) d\theta + e \left[\text{Sup}_{i \in K} \int_{\Theta} l(p, \theta) \Pr(\theta|D,i) d\theta \right] \quad (14)$$

The interpretation of (14) is as follows. As before, the policymaker has an estimate of the true distribution but he does not know the true parameter values in (1)-(3) nor the true structure of the economy. Now, however, he is reluctant to make a decision based on a single distribution estimate. Thus, he assigns a probability e that the true distribution (of the parameters) might be different from his estimate. To account for this uncertainty the policymaker minimises the worst possible loss. Thus, e reflects the degree of ambiguity aversion. In the limiting case that $e=1$ he sets policy only using a minimax rule. When $e=0$, (14) reduces to a standard Bayesian expected loss function. In intermediate cases, the policy will give a larger weight to the worst possible scenario.

Specifically, defining r_i^0 as the optimal rule for the case of only parameter and structural risk and r_i^1 as the rule that minimises the worst-case scenario, the solution for intermediate degrees of ambiguity aversion is

$$r_i = e r_i^0 + (1-e) r_i^1 \quad (15)$$

Since I have already found r_i^0 above (equation 13) I only need to find the minimax rule.

4.1. Minimax solution

To obtain the minimax solution I follow Brock *et al*'s (2003) local robustness analysis. Specifically, I am looking for a policy rule that minimises the expect-

ed loss, given that an adversarial agent will, starting from fixed parameters in (9)-(10), select one or more parameters from a given interval to hurt the policymaker as much as possible.⁹

I consider parameter uncertainty in the model (which I denote i^*) that includes the lagged output gap, the lagged inflation gap, and *all* other exogenous variables. Further, I consider uncertainty around a subset of parameters¹⁰

$$\theta^i = (a_1, a_j, b_1, b_2, b_j, \sigma_{b_2}^2, \sigma_{a_1}^2) \text{ for } j = 3 \dots n$$

Although I focus on estimation issues later, for expositional continuity I present the minimax solution here. In particular, I obtain (see section 5.2) that the maximum loss occurs when

$$\theta^{i^*} = (\underline{a_1}, \overline{a_j}, \underline{b_1}, \underline{b_2}, \overline{b_j}, \overline{\sigma_{b_2}^2}, \overline{\sigma_{a_1}^2}) \text{ for } j = 3 \dots n$$

where a lower/upper bar indicate that the loss function is maximised when the parameter is at the minimum/maximum of the boundaries considered (i.e. the estimated value among all the models considered).

It follows that the optimal minimax rule is¹¹

$$r_i^i = \frac{\kappa_{2,i^*} (\pi_i - \pi^*) + \left(\kappa_{1,i^*} \kappa_{2,i^*} - \frac{\omega_{1,i^*}}{2} \right) y_i + \sum_{j=3}^n \left(\kappa_{2,i^*} \kappa_{j,i^*} - \frac{\omega_{2,i^*,j}}{2} \right) x_{i,i^*,j}}{\kappa_{2,i^*}^2 + \sigma_{k_{2,i^*}}^2} \quad (16)$$

with $\kappa_{2,i^*} = \underline{a_1} \underline{b_2}$, $\kappa_{1,i^*} = \underline{a_1} (1 + b_1)$, $\kappa_{j,i^*} \equiv \overline{a_j} (1 + c_j) + \underline{a_1} \overline{b_j}$, $\sigma_{k_{2,i^*}}^2 \equiv \underline{a_1}^2 \overline{\sigma_{b_2}^2} + \overline{\sigma_{a_1}^2} \underline{b_2}^2 + \overline{\sigma_{a_1}^2} \overline{\sigma_{b_2}^2}$

The intuition of these results is the following. First, a reduction in a_1 and b_2 makes the instrument less powerful and thus increases the policymaker's loss. Second, an increase in b_1 or $|b_j|$ moves output away from potential, which in turn deviates inflation from target. Compared to the optimal rule under no Knightian uncertainty, the adjustments due to changes in a_1 , b_2 , b_1 , and $|b_j|$ are ambiguous, depending upon the relative uncertainty over the mean and variance of the coefficients. Third, an increase in the parameter makes the policymaker more vulnerable to changes in the exogenous variables. To deal with uncertainty over this parameter he responds more aggressively to $x_{i,j}$. Finally, because the policymaker is risk averse, an increase in the policy instrument's parameter variance will harm him and will lead to a more cautious response.¹²

From this exercise it is clear that the effect of uncertainty on the optimal response is ambiguous. In contrast to recent findings (e.g. Hansen and Sargent 2003, Giannoni 2002, 2006) regarding the effects of unknown parameters' distribution (a larger optimal feedback rule), Brock et al. (2003) show

that whether uncertainty increases or decreases the optimal policy response depends on which parameters have an uncertain distribution and on the interaction among the parameters. What our exercise demonstrates is the importance of the parameters' risk (in the sense of the variance of the coefficients) in determining such interaction.

5. MODEL ESTIMATION

To motivate future empirical work I present a simple exercise where the vector x includes one additional lag of the output gap (y_{t-1}) and one of the Fed funds rate (r_{t-1}), which gives a total of four models. Further, to provide a more reasonable account of the Fed's behaviour I generalise the core model by including the output gap into the objective function. The analytical solution for the optimal rules under this setup is presented in the appendix.

I estimate the model (1)-(3) using annual U.S. data over the period 1970-2003. Inflation is defined as the average four quarters change in the GDP chain-weighted price index; the output gap is defined as the (four quarters average) percentage deviation of real GDP to potential GDP as estimated by the congressional budget office; and the Fed funds rate is the average rate during year t . Before proceeding with the results I present some methodological issues.

5.1. Bayesian model averaging methodology

To apply Bayesian model averaging we need an estimate of the posterior probability for each model and for the parameters vector. For a particular model i its posterior probability is given by

$$\Pr(i|D) = \frac{\Pr(D|i)\Pr(i)}{\sum_{k=1}^K \Pr(D|k)\Pr(k)} \quad (17)$$

where $\Pr(D|i) = \int \Pr(D|\theta, i)\Pr(\theta|i)d\theta$ is the integrated likelihood of model i , $\Pr(\theta|i)$ is the prior density of θ under model i , $\Pr(D|\theta, i)$ is the likelihood, and $\Pr(i)$ is the prior probability of model i .

To estimate (17) two priors have to be specified, one for the model and another for the parameter's vector. I take a pragmatic approach. First, I assume a non-informative (uniform) prior over the model space. Thus, the policymaker assigns a prior probability of 0.5 to the outcome that the coefficients attached to the j exogenous variables are different from zero. With this assumption at hand, I follow an approximation suggested by Raftery (1995) to calculate the posterior probability for each model. Specifically, the weights are BIC adjusted likelihoods

$$\Pr(i|D) \approx \frac{e^{(-0.5 \cdot BIC_i)}}{\sum_i e^{(-0.5 \cdot BIC_i)}} \quad (18)$$

Table 1: Optimal rules

	Model 1	Model 2 (r_{t-1})*	Model 3 (y_{t-1})*	Model 4 (r_{t-1} & y_{t-1})*	
<i>Model posterior</i>	0.2420	0.2472	0.2573	0.2535	
<i>Certainty equivalence optimal rule ($\lambda=0.5$)</i>					Average
Inflation gap	1.0683	0.2901	1.7704	0.8432	0.9930
Output gap	1.9537	3.3350	3.2529	2.6919	2.8084
r_{t-1}		-3.3249		-0.1390	-1.7319
y_{t-1}			-2.2193	-1.6685	-1.9439
<i>Certainty equivalence optimal rule ($\lambda=1$)</i>					Average
Inflation gap	0.5496	0.1451	0.9688	0.4304	0.5235
Output gap	1.8642	3.3330	2.9939	2.6317	2.7057
r_{t-1}		-3.2688		-0.0031	-1.6360
y_{t-1}			-1.8909	-1.5471	-1.7190
<i>e=1 optimal rule</i>					
	$\lambda = 0.5$	$\lambda = 1$			
Inflation gap	0.0410	0.0359			
Output gap	0.2610	0.1892			
r_{t-1}	-1.4533	-1.6712			
y_{t-1}	-0.2273	-0.7888			
<i>e=0 optimal rule</i>					
	$\lambda = 0.5$	$\lambda = 1$			
Inflation gap	0.7860	0.4998			
Output gap	1.4207	1.7745			
r_{t-1}	-0.6185	-0.2979			
y_{t-1}	-1.2518	-1.3300			
<i>e=0.5 optimal rule</i>					
	$\lambda = 0.5$	$\lambda = 1$			
Inflation gap	0.4135	0.2679			
Output gap	0.8409	0.9819			
r_{t-1}	-1.108	-0.984			
y_{t-1}	-0.7380	-1.0594			

*Refers to the additional variables added to the model

Similarly, I impose a uniform distribution to the coefficients and I assume that the errors are i.i.d. normal with known variance. As explained in Brock *et al* (2003), this assumption has the advantage that the posterior estimates of the mean and variance of the regression coefficients are given by the OLS esti-

mates. Thus, I estimate the IS and the Phillips curves separately by OLS (estimates reported in Appendix 2), I obtain the posterior probability for each, and then average the weights obtained for each model. For example, with two models, if model i has a weight of 0.6 for the IS curve and 0.5 for the Phillips curve while model l has a weight of 0.4 and 0.5, then I would use *model* posterior weights of 0.55 for i and 0.45 for l .

5.2. Minimax methodology

As mentioned above, the optimal minimax rule ($e=1$) that I consider is the one that minimises the expected loss of the worst possible scenario in the model that includes all the exogenous variables in the vector x (in this exercise the vector x includes one additional lag of the output gap (y_{t-1}) and one of the Fed funds rate (r_{t-1})). Specifically, embedding the generic optimal rule for this model (given in (A.3) in the appendix) in the loss function (A.1) I ask: what parameter configuration maximises the loss?

To answer this question I first obtain, for each parameter, the coefficient estimates for the four different models. Then, given these values, I evaluate the loss function using all possible permutations of the set of coefficients. As mentioned above, I obtain that the maximum loss is given when

$$\theta^{j*} = \left(\underline{a}_1, \overline{a}_1, \underline{b}_1, \overline{b}_1, \underline{b}_2, \overline{b}_2, \overline{\sigma}_{a_1}^2 \right) \text{ for } j = 3, 4$$

where a lower/upper bar indicates that the parameter that maximises the loss function is the one obtained in the model with the minimum/maximum coefficient estimate. The minimax rule is the one that uses these estimates as if they were the true values.

5.3. Results

Table 1 presents the certainty equivalent optimal rules for each model (and for two different weights on the output gap), the optimal rule under $e=1$ for the model with two additional variables, the rule under no Knightian uncertainty ($e=0$), and a rule for an intermediate degree of ambiguity ($e=0.5$).¹⁴ I am not placing any particular emphasis on the absolute values of the rules, which would be expected to vary with a number of factors (the model space, prior specification, the boundaries selected, and so on); rather, the overall pattern obtained is of interest.

In the minimax rule the optimal response to the output and the inflation gaps is much smaller than the average certainty equivalent rule (around 85 to 95 percent lower). This is due to very small mean estimates in one of the models of the instrument's response coefficients a_1 and b_2 , and relatively large estimated variances. By looking at (16) one can see that this is exactly the condition necessary for uncertainty to lead to an extra precautionary motive. Second, in contrast to previous research (eg. Estrella and Mishkin, 1999; Rudebusch, 2001), I find a quite large adjustment due parameter and struc-

tural risks. Specifically, the optimal rule for $e=0$ gives a response to the inflation gap that is around 20 percent (for $\lambda=0.5$) and 4 percent (for $\lambda=1$) lower than the average certainty equivalence rule. The response to the output gap shows an even larger reduction; around 50 percent for $\lambda=0.5$ and 35 percent for $\lambda=1$. Finally, the additional lags of the output gap and the Fed funds rate show similar reductions.

Since both cases ($e=0$ and $e=1$) provide lower responses than the certainty equivalent rule, it follows directly that the average also results in lower response coefficients. It is appealing that the reaction functions under $e=0$ and $e=0.5$ show values close to those suggested by Taylor (1993). As argued before, however, I do not want to draw any major conclusion from the absolute values obtained.

6. CONCLUSION

This paper solves the problem of a regulator who fears model misspecification and is reluctant to assign a unique distribution to the model(s) parameters. As expected from a standard Bayesian perspective I find that model risk leads the monetary authority to be more cautious. Probably more surprising is the finding that Knightian uncertainty also leads to a precautionary motive. This result, however, should be corroborated with a more complete empirical study that considers a wider model space (for example with different assumptions on expectations formation), different priors, and other specifications that would make the optimal rule realistically robust to uncertainty.

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APPENDIX 1

Policy rules with output in the objective function

In this appendix I generalise the model with the assumption that, in addition to inflation, the policymaker includes the output gap in the objective function. Thus, following Estrella and Mishkin (1999) the policymaker minimises for each model i

$$E_t(\pi_{t+2,i} - \pi^*)^2 + \lambda E_t(y_{t+1,i})^2 \quad (A1)$$

where λ is the weight attached to the output gap.

Using the expected values and the variances of inflation and the output gap given by the structural model in (1)-(3), I look for the rule that minimises this loss function under three alternative scenarios that depend on the degree of risk and uncertainty that the policymaker faces.

The certainty equivalence rule is obtained by assuming that the policymaker knows (or believes) that model \hat{i} is the true model of the economy and he also knows the true parameter values:

$$r_i = \frac{a_{1,i}(\pi_i - \pi^*) + (\kappa_{1,i}a_{1,i} + b_{1,i}\lambda)y_i + \sum_{j=3}^n (a_{1,j}\kappa_{j,i} + \lambda b_{j,i})x_{i,j}}{b_{2,i}(\lambda + a_{1,i}^2)} \quad (A2)$$

The *generic* minimax rule ($e=1$) (the one that minimises the loss function of the model with all the exogenous variables — i^* —) is essentially an extension of the results obtained by Estrella and Mishkin (1999) who consider parameter risk in a single model that has one exogenous variable:

$$r_i = \frac{\left[\begin{aligned} &\kappa_{2,i^*}(\pi_i - \pi^*) + \left(\kappa_{1,i^*}\kappa_{2,i^*} + b_{1,i^*}b_{2,i^*}\lambda - \frac{\omega_{1,i^*}}{2} \right) y_i + \\ &+ \sum_{j=3}^n \left(\kappa_{2,i^*}\kappa_{j,i^*} + \lambda b_{2,i^*} \sum_{j=3}^n b_{j,i^*} - \frac{\omega_{2,i^*,j}}{2} \right) x_{i^*,j} \end{aligned} \right]}{\kappa_{2,i^*}^2 + \sigma_{k_{2,i^*}}^2 + \sigma_{b_{2,i^*}}^2 + \lambda b_{2,i^*}^2} \quad (A3)$$

Finally, the rule that minimises the loss function without Knightian uncertainty but that incorporates both structural and parameter uncertainty ($e=0$) is:

$$r_i = \frac{\sum_{i=1}^K \Pr(i|y) \left[\begin{aligned} &\kappa_{2,i}(\pi_i - \pi^*) + \left(\kappa_{1,i}\kappa_{2,i} + b_{1,i}b_{2,i}\lambda - \frac{\omega_{1,i}}{2} \right) y_i + \\ &+ \sum_{j=3}^n \left(\kappa_{2,i}\kappa_{j,i} + \lambda b_{2,i} \sum_{j=3}^n b_{j,i} - \frac{\omega_{2,i,j}}{2} \right) x_{i,j} \end{aligned} \right]}{\sum_{i=1}^K \Pr(i|y) \left(\kappa_{2,i}^2 + \sigma_{k_{2,i}}^2 + \sigma_{b_{2,i}}^2 + \lambda b_{2,i}^2 + (\bar{\kappa}_2 - \kappa_{2,i})^2 + \lambda (\bar{b}_2 - b_{2,i})^2 \right)} \quad (A4)$$

APPENDIX 2

**OLS coefficient estimates used in the calculation
of the optimal policy rules**

	<i>Model 1</i>	<i>Model 2</i> (r_{t-1})*	<i>Model 3</i> (y_{t-1})*	<i>Model 4</i> (r_{t-1} & y_{t-1})*
<i>a1</i>	0.1727	0.0143	0.3231	0.1459
<i>a3</i>		-0.2146		-0.1825
<i>a4</i>			-0.2558	-0.1835
<i>b1</i>	0.5397	0.3276	0.8095	0.8525
<i>b2</i>	0.3050	0.0984	0.3019	0.3300
<i>b3</i>		-0.3160		0.0460
<i>b4</i>			-0.4511	-0.4714
<i>c3</i>		0.8030		0.8030
<i>c4</i>			0.6015	0.6015
<i>var a1</i>	0.0114	0.0131	0.0159	0.0204
<i>var b2</i>	0.0081	0.0261	0.0061	0.0286
<i>var b1</i>	0.0169	0.0361	0.0196	0.0576
<i>cov (b1, b2)</i>	0.0034	-0.0174	0.0012	-0.0309
<i>cov (b1, b3)</i>		0.0288		0.0493
<i>cov (b1, b4)</i>			-0.0116	-0.0338
<i>cov (b2, b3)</i>		-0.0280		-0.0343
<i>cov (b2, b4)</i>			-0.0001	0.0150

*Refers to the additional variables added to the model

ENDNOTES

1. Correspondence address: Clarkson University,, PO Box 5790, Potsdam, New York 13699. Phone: 315-268-3870, E-mail: dnocetti@clarkson.edu I wish to thank an anonymous referee, seminar participants at the University of Memphis, and specially Bill Smith, for many helpful comments.

2. Following Knight (1921) I refer to uncertainty as a situation in which the decision maker has an unknown probability distribution of the outcomes and to risk as a situation in which the true distribution is well identified. Further, I differentiate between structural risk and parameter risk. The former refers to cases in which the policymaker does not know the true specification of the economy (for example, he might consider models that have different lags or different assumptions about expectations formation) while the latter is used exclusively for risk about the coefficients of the structural models. An equivalent terminology is used for structural and parameter uncertainty.

3. Min and Zellner (1993) show that Bayesian Model Averaging is optimal in the sense that it minimises the expected forecasts errors.
4. Brock *et al.* (2003, 2004) provide a complete description of this technique in the context of monetary policy.
5. Svensson (1997) shows that the solution to the period by period optimisation problem is equivalent to the dynamic solution of two period's ahead inflation forecasts. For the empirical estimation I generalise this setup by including output in the objective function.
6. Note, however, that this result depends on the magnitude of the covariance estimates.
7. See also Gilboa and Schmeidler (1989) for an axiomatic treatment of decision making under Knightian uncertainty
8. Brock *et al.* (2003) suggest this specification but they do not pursue the problem further.
9. This is different from the approach employed by Giannoni (2002, 2006) where the policymaker considers an interval in the coefficients of (1)-(3). Because the interval is known and constant, the variance of the parameters would not appear in the optimal policy rule (i.e. the optimal rule is the same as the certainty equivalence rule using the parameters that maximise the loss).
10. These are the most important parameters in terms of the feedback rule and the loss function. Since the persistence parameter of the exogenous variables, c_j , does not change across models I do not consider it for the minimax analysis.
11. The general form of the rule is essentially an extension of the results obtained by Estrella and Mishkin (1999) who consider multiple parameter risk in a single model with one exogenous variable.
12. Many of these results are not general but the product of the numerical estimation. The effects on the loss function, however, are in line with previous research and economic intuition.
13. All data were obtained from the Federal Reserve Bank of St. Louis website.
14. The estimated parameters are presented in the appendix.

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