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A collective tournament under (un)limited liability

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ABSTRACT

In the principal-agent literature, a collective tournament, i.e. a tournament between teams, has been proposed as a solution to the free-rider problem. Competition between the teams is said to foster within-team cooperation and, hence, to mitigate free-riding. In this paper, we analyse the impact of an agent's liability on the tournament outcome. In the more realistic case of limited liability, a collective tournament is found to perform very poorly. Free-riding is, in this case, even intensified when applying a collective tournament.

1. INTRODUCTION

A COLLECTIVE TOURNAMENT, i.e. a tournament between teams, has been proposed as a solution to the free-rider problem. Competition between several teams is said to foster within-team cooperation and, hence, to mitigate free-riding. This argument is supported by a theoretical paper of Drago *et al.* (1996) who show that, for a certain, exogenously given prize structure, a collective tournament can induce first-best efforts. Further, in an experimental study, Nalbantian and Schotter (1997) compare a collective tournament to other group incentive schemes. They find that collective tournaments lead to the highest mean outputs and so should be preferred, at least by risk-neutral principals.

It is straightforward to find examples of collective tournaments in practice: the reconstruction of the World Trade Centre e.g. was started by a tender procedure, which induced several teams of architects to submit proposed designs for the new building. Finally, the proposal of an architect team from Berlin was selected. The selection process had all the characteristics of a collective tournament. Several teams were in competition for a prize (here the fame and monetary gain from designing the building) and they spent effort and other resources while developing the proposals. Similarly, at universities it is often the case that resources are allocated to departments according to their performance. This allocation process could again be interpreted as a tournament between the departments of a university.

In the literature on moral hazard,[2] it is argued that an agent's liability may play an important role, when determining the relative quality of different incentive devices. An agent with limited liability usually receives a rent, i.e. his payoff exceeds his reservation utility. As a consequence, the principal does not only want to provide an incentive to the agent, he also wants to keep the agent's rent as small as possible. This might yield distortions influencing the relative appropriateness of different incentive schemes.

The aim of this paper is thus to assess the appropriateness of collective tournaments in both cases, where agents have either unlimited or limited liability. In the former case, the first-best solution will be achieved, hence the free-rider problem will be completely eliminated. The principal 'finances' the first-best solution by demanding an entrance fee from the agents, that is, by setting a negative loser-prize. Therefore, he has to change the tournament parameters, when we turn to the case of limited liability, where the agents are no longer able to afford an entrance fee. In this case, a collective tournament is a very poor incentive device. In order to motivate the agents, the principal has to install a very high winner-prize. This is not worthwhile for him, so both the winner-prize and the agents' efforts are rather small. Compared to a simple piece-rate model, the collective tournament even intensifies the free-rider problem instead of mitigating it.

This paper is closely related to the literature on group rent seeking contests, where groups compete for an exogenously given rent. This literature can be roughly divided into two strands. Examples for the first strand include Nitzan (1991), Lee and Kang (1998), Davis and Reilly (1999) or Gürtler (2005). These papers assume that individual outlays (which correspond to effort in the current model) are observable within groups. As a consequence, the groups implement sharing-rules for the rent so that, in case of successful inter-group competition, an individual's share in the rent is increasing in the outlay he has chosen. In this way, the groups may alleviate the free-rider problem. The second strand of literature (see e.g Müller and Wärneryd (2001) or Konrad (2004)) assumes that a successful inter-group contest is followed by an intra-group contest, in which the group members spend resources in order to receive a higher fraction of the rent. In a symmetric situation, this does not affect free-riding in the inter-group contest, as, in equilibrium, the rent is equally divided between group members. In an asymmetric situation, however, the free-rider problem may become more or less severe. The current paper abstracts from possibilities to mitigate the free-rider problem by employing intra-group allocation mechanisms. Instead, how a principal might affect the free-rider problem by appropriately determining the tournament prizes is analysed. This should be of high importance e.g. in labour relations, whereas intra-group allocation rules should play a more essential rule in political lobbying.

The paper is organised as follows: the next section introduces the basic model. Section 3 contains two benchmark cases, the first-best solution and a simple piece-rate model, demonstrating the relevance of the free-rider problem. The tournament solution is presented in section 4. Section 5 offers several model extensions, while section 6 concludes.

2. THE BASIC MODEL

Consider a risk-neutral principal arranging a tournament between two teams, each consisting of n homogeneous, risk-neutral agents. Let w_s denote the winner-prize and l_s the loser-prize.[3] The prize a team receives is assumed to be equally divided between the team's members.

Agent j (j=1,…,n) of team i (i=1,2) exerts unobservable effort e_{ji} at cost $C(e_{ji})=(e_{ji})^{\delta}$, with $\delta > 1$.[4] The parameter δ determines the convexity of the effort cost function. Efforts lead to (contractible) team output according to the subsequent concave function:

$$y_i = \left(\sum_{j=1}^{n} e_{ji}\right)^{\gamma}, \ \gamma \leq 1 \quad .^5 \tag{1}$$

Following the contest literature (see e.g. Skaperdas (1996), Gradstein and Konrad (1999) or Huck *et al.* (2001)), we assume that team 1's contest success function, i.e. its probability of winning is given by:

$$P_1 = \begin{cases} \dfrac{\left(\sum_{j=1}^{n} e_{j1}\right)^{\gamma}}{\left(\sum_{j=1}^{n} e_{j1}\right)^{\gamma} + \left(\sum_{j=1}^{n} e_{j2}\right)^{\gamma}} & \text{for } \left(\sum_{j=1}^{n} e_{j1}\right)^{\gamma} + \left(\sum_{j=1}^{n} e_{j2}\right)^{\gamma} > 0 \text{ and} \\ 0.5 & \text{otherwise.} \end{cases} \tag{2}$$

The principal first determines the prize structure such that aggregate output net off wage costs, $y_1 + y_2 - w_s - l_s$, is maximised.[6] Thereafter, agent j of team i chooses effort to maximise $\frac{w_s - l_s}{n} \cdot P_i + \frac{l_s}{n} - C(e_{ji})$. Finally, each agent is supposed to possess an outside option, yielding zero utility. Hence, an agent will only take part in the tournament, if this leads to expected utility of zero or more.

3. THE BENCHMARK CASES

We start by considering two benchmark cases, the first-best solution and a simple piece-rate model. Both are helpful. The former represents a situation, where the free-rider problem is completely solved, whereas the latter shows nicely the relevance of free-riding.

In the first-best solution, aggregate output minus total effort costs is maximised. The resulting maximisation problem is given by (3), the corresponding first-order conditions by (4):[7]

$$\sum_{i=1}^{2}\left(\sum_{j=1}^{n} e_{ji}\right)^{\gamma} - \sum_{i,j}\left(e_{ji}\right)^{\delta} \rightarrow \underset{e_{ji}}{Max!} \tag{3}$$

$$\gamma\cdot\left(\sum_{j=1}^{n} e_{ji}\right)^{\gamma-1} = \delta\cdot\left(e_{ji}\right)^{\delta-1} \tag{4}$$

As the effort cost function is the same for all agents, first-best efforts are symmetric and satisfy

$$e_{fb} = \left(\frac{\gamma}{\delta}\cdot n^{\gamma-1}\right)^{\frac{1}{\delta-\gamma}}$$

Under piece-rates, each agent receives a fixed wage f_s and a fraction $\alpha_s \in [0;1]$ of output produced by his team. The principal optimally determines these parameters, and, thereafter, the agents choose simultaneously their optimal efforts. Using backward induction, we first consider the maximisation problem of agent *j* of team *i*. This agent maximizes

$$f_s + \alpha_s\cdot\left(\sum_{j=1}^{n} e_{ji}\right)^{\gamma} - \left(e_{ji}\right)^{\delta} \rightarrow \underset{e_{ji}}{Max!} \tag{5}$$

The first-order condition is

$$\alpha_s\cdot\gamma\cdot\left(\sum_{j=1}^{n} e_{ji}\right)^{\gamma-1} = \delta\cdot\left(e_{ji}\right)^{\delta-1} \tag{6}$$

Again, we get a symmetric solution with effort given by

$$e_{pr,s} = \left(\alpha_s\right)^{\frac{1}{\delta-\gamma}}\cdot e_{fb}$$

Note that the first-best will only be achieved, if $\alpha_s = 1$. Otherwise, i.e. if $\alpha_s < 1$, efforts will be inefficiently low, as the agents start to free-ride. If $\alpha_s < 1$, an agent will receive only part of his marginal product, while bearing the complete effort costs. Hence, he undersupplies effort with respect to the first-best level. Naturally, how intense free-riding is depends on the principal's choice of α_s. A lower α_s yields lower effort and, hence, intensifies the free-rider problem. In order to assess precisely the relevance of the free-rider problem under the piece-rate scheme, the maximisation problem of the principal has to be solved. Doing this, the case of unlimited and limited liability are treated separately. We start with the former case, where the principal's maximisation problem is

$$\begin{aligned}&\underset{\alpha_u, e_{pr,u}}{Max}\ (1-\alpha_u \cdot n)\cdot(n\cdot e_{pr,u})^{\gamma} - f_u \cdot n\\ &s.t.:\ \frac{\alpha_u \cdot \gamma \cdot (n)^{\gamma-1}}{\delta} = (e_{pr,u})^{\delta-\gamma},\\ &\quad f_u + \alpha_u \cdot (n\cdot e_{pr,u})^{\gamma} - (e_{pr,u})^{\delta} \geq 0.\end{aligned} \tag{7}$$

Maximisation problem (7) has the following solution:[8]

$$f_u = \left(\frac{\gamma}{\delta}\right)^{\frac{\gamma}{\delta-\gamma}} \cdot \left(\frac{\gamma}{\delta}\cdot n^{\frac{\delta\cdot(\gamma-1)}{\delta-\gamma}} - n^{\frac{\gamma\cdot(\delta-1)}{\delta-\gamma}}\right) < 0, \alpha_u = 1, e_{pr,u} = e_{fb} \tag{8}$$

It is not surprising that, under unlimited liability, the piece-rate scheme completely eliminates free-riding. The principal maximises the achievable surplus by setting $\alpha_u = 1$ and completely extracts this surplus by demanding an entrance fee from the agents, i.e. by setting a negative fixed wage. In other words, he uses some kind of 'sell-the-shop' contract to achieve efficiency.

Under limited liability, the principal is forced to change the wage contract, as he is no longer allowed to charge an entrance fee. Technically, the principal has to consider as a further constraint that $f_1 \geq 0$. The solution to this extended maximisation problem is given by

$$f_l = 0, \alpha_l = \frac{\gamma}{\delta\cdot n}, e_{pr,l} = \left(\frac{\gamma^2 \cdot n^{\gamma-2}}{\delta^2}\right)^{\frac{1}{\delta-\gamma}} \tag{9}$$

With limitedly liable agents, the first-best is no longer achieved. As the principal is forbidden from charging an entrance fee, he extracts a surplus by lowering the piece-rate, that is by choosing $\alpha_1 < 1$. As mentioned before, this results in free-riding since an agent's marginal product exceeds his marginal compensation. Further, note that free-riding becomes more intense the more agents are in a team. A *ceteris paribus* increase in n lowers each agent's share in output and, hence, equilibrium effort.

Before turning to the next section, it is worth mentioning a mechanism, introduced by Holmström (1982), which yields efficiency even under limited liability. The principal might offer the agents a so-called budget-breaking contract that will compensate them for their effort costs, only if the efficient output level is reached. Otherwise, the agents receive a payment of zero. In this case, there exists an equilibrium with all agents providing efficient effort. Yet, contrary to the tournament or the piece-rate scheme, the proposed mechanism is hardly used in practice, so its practical implementation might entail high costs. Most importantly, the principal might be interested in sabotaging the team or in colluding with one of the team's members.[9] In this way, the principal can save on almost complete wage costs, while losing only a small fraction of output. In what follows, we assume that these problems are so severe that the budget-breaking mechanism is dominated by the tournament and piece-rate scheme.[10]

To summarise this section, in the realistic case of limited liability, a piece-rate scheme does not solve the free-rider problem. Clearly, the question arises whether a collective tournament does better in motivating the agents. This is analysed next.

4. Solution to the tournament

Using backward induction, we start by determining the agents' efforts for given tournament prizes. Agent *j* of team 1 chooses effort to maximise

$$\frac{w_s - l_s}{n} \cdot \frac{\left(\sum_{j=1}^{n} e_{j1}\right)^{\gamma}}{\left(\sum_{j=1}^{n} e_{j1}\right)^{\gamma} + \left(\sum_{j=1}^{n} e_{j2}\right)^{\gamma}} + \frac{l_s}{n} - \left(e_{j1}\right)^{\delta}, \tag{10}$$

yielding the following first-order condition:

$$\frac{w_s - l_s}{n} \cdot \frac{\gamma \cdot \left(\sum_{j=1}^{n} e_{j1}\right)^{\gamma-1} \cdot \left(\sum_{j=1}^{n} e_{j2}\right)^{\gamma}}{\left[\left(\sum_{j=1}^{n} e_{j1}\right)^{\gamma} + \left(\sum_{j=1}^{n} e_{j2}\right)^{\gamma}\right]^2} = \delta \cdot \left(e_{j1}\right)^{\delta-1} \tag{11}$$

It is easy to see that the solution is symmetric, i.e. all members of the first team choose same effort $e_{1,s}$. Analogously, all members of team 2 exert same effort $e_{2,s}$. Further, one can show that efforts are symmetric across teams, hence $e_{1,s} = e_{2,s} = e_s$, with e_s given by

$$e_s = \left(\frac{\left(w_s - l_s\right) \cdot \gamma}{4 \cdot n^2 \cdot \delta}\right)^{\frac{1}{\delta}} \tag{12}$$

Optimal effort is increasing in the prize difference w_s - l_s, the productivity parameter γ and decreasing in the cost parameter δ and the number of agents per team. As with the piece-rate scheme, the more members a team consists of, the lower is the optimal effort.

In analogy to the piece-rate model, we differ between the case of unlimited and limited liability of the agents, when deriving the optimal prize structure. Section 4.1 deals with the former case, section 4.2 with the latter.

4.1 The principal's decision under unlimited liability

Under unlimited liability, the principal's maximisation problem is

$$\begin{aligned} &\underset{w_u, l_u, e_u}{Max}\ 2\cdot(n\cdot e_u)^{\gamma} - w_u - l_u \\ &s.t.: \frac{(w_u - l_u)\cdot\gamma}{4\cdot n^2\cdot\delta} = (e_u)^{\delta}, \\ &\frac{w_u + l_u}{2\cdot n} - (e_u)^{\delta} \geq 0. \end{aligned} \tag{13}$$

The solution to problem (13) is given by (14) and immediately leads to Proposition 1:

$$w_u = \left(\frac{2\cdot n^2\cdot\delta}{\gamma} + n\right)\cdot(e_u)^{\delta}\ , \quad l_u = \left(-\frac{2\cdot n^2\cdot\delta}{\gamma} + n\right)\cdot(e_u)^{\delta}\ , \quad e_u = \left(\frac{n^{\gamma-1}\cdot\gamma}{\delta}\right)^{\frac{1}{\delta-\gamma}} \tag{14}$$

Proposition 1. Under unlimited liability, a tournament between two teams completely eliminates the free-rider problem.

As expected, the collective tournament achieves the first-best solution when agents have unlimited liability. From (14), the sum of the two tournament-prizes can be shown to equal $2\cdot n\cdot(e_u)^{\delta}$, i.e. the principal determines the two prizes such that each agent receives his reservation utility. Thus, the principal again receives the complete rent produced by the agents. Maximising this rent by appropriately determining the prize structure naturally leads to first-best efforts. Note that l_u is negative. This implies that the principal will have to change the prize structure if we impose a limited liability constraint. The solution in this more realistic case is derived next.

4.2 The principal's decision under limited liability

If the agents neither have any wealth nor are liable, the principal's maximisation problem in (13) will be extended by the additional constraint w_1, $l_1 \geq 0$. This extended problem has the following solution:

$$w_l = \left(\frac{4\cdot n^2\cdot\delta}{\gamma}\right)\cdot(e_l)^{\delta}\ , \quad l_l = 0\ , \quad e_l = \left(\frac{n^{\gamma-2}\cdot\gamma^2}{2\cdot\delta^2}\right)^{\frac{1}{\delta-\gamma}} \tag{15}$$

Under limited liability, a collective tournament does no longer lead to the first-best solution. On the contrary, it performs very poorly, as the subsequent proposition shows:

Proposition 2. Under limited liability, the collective tournament intensifies the free-rider problem compared to the piece-rate scheme.

Proposition 2 shows that, in the more realistic case of limited liability, the collective tournament induces no further incentives to work hard, it destroys existing ones and so intensifies the free-rider problem. This finding seems to be at odds with the practical prevalence of collective tournaments and so requires further explanation. Let us therefore transform (12), in combination with l_1, to obtain

$$w_l = \frac{4 \cdot n^2 \cdot \delta}{\gamma} \cdot (e_l)^{\delta} \tag{12a}$$

We see that the winner-prize w_1 is many times greater than the aggregate costs, entailed by equilibrium effort, $2 \cdot n \cdot C(e_l) = 2 \cdot n \cdot (e_l)^{\delta}$. Thus, incentive setting is very expensive for the principal. To induce a relatively high effort level, the principal has to pay an extremely high winner-prize. Hence, he prefers to save on wage costs by choosing a rather low winner-prize and to tolerate free-riding.[11]

5. Extensions of the basic model

Before turning to the model extensions, note that the prize-setting stage of the tournament is simply a special case of a Nash bargaining game, with the principal having complete bargaining power.[12] Technically, introducing the limited liability constraint restricts the set of possible agreements, and hence the set of achievable utility pairs, to those with a non-negative loser-prize. As a result, the bargaining outcome under unlimited liability is no longer available, and the principal chooses a point with a lower prize-spread.

In this section, we restrict our attention to the limited liability case and analyse three extensions of the basic model that all change the structure of the bargaining game. First, we ask whether a change in the bargaining power yields an increase in efficiency, i.e. we consider a situation where the agents make a take-it-or-leave-it offer to the principal. Second, we alter the disagreement point. In particular, we assume that an agent's reservation utility becomes strictly positive. Finally, we analyse how a change in the principal's objective function affects the tournament results. In science, for example, it is often the highest output that is of interest. Hence, we consider a variation of the basic model, where the principal maximises $\text{Max}\{y_1, y_2\} - w_l - l_l$. Referring to the bargaining process, such a change in the principal's objective function leads to a different set of utility pairs to be realized through negotiation.

5.1 Reversed bargaining power

In the case of reversed bargaining power, the principal faces a competitive labour market and so is constrained to make zero profit.[13] Under this constraint, he chooses the tournament-prizes such that agents' utilities are maximised. In the optimum, the first-best solution is again not achieved, as this solution would require a negative loser-prize. Yet, efforts are higher than efforts derived in section 4, i.e. reversed bargaining power mitigates the free-rider problem in collective tournaments. This is intuitive. Under reversed bargaining power, the principal again sets the loser-prize equal to zero. While in the previous section he was allowed to receive a rent, he is now constrained to make zero profit. Hence, he chooses a higher winner-prize than in section 4 (which induces higher efforts), although the additional wage costs exceed the additional output produced by the two teams.

5.2 Agents with strictly positive reservation utility

If an agent has a strictly positive reservation utility $\bar{U} > 0$, his participation constraint will change to $\frac{w_l + l_l}{2 \cdot n} - (e_l)^{\delta} \geq \bar{U}$. The solution to the principal's maximisation problem then depends on the value of $\bar{U}$. In fact, there are three possible equilibria: In case (i), $\bar{U}$ is relatively small and the limited-liability constraint is the only binding constraint. For intermediate values of $\bar{U}$, i.e. in case (ii), both constraints are binding. Finally, in case (iii), $\bar{U}$ is large so the limited liability constraint is slack, while the participation constraint is not.[14]

(i) In the case of a small, but positive reservation utility, the results from section 4 do not change. An agent's expected utility still exceeds his reservation utility. Consequently, the participation constraint remains slack and it is optimal to set the loser-prize equal to zero. Clearly, the free-rider problem is as significant as before.

(ii) For intermediate values of $\bar{U}$, both constraints become binding. In this case, efforts increase compared to the case, where $\bar{U} = 0$ so that the free-rider problem becomes less severe. The intuition for this result is as follows: Let $\bar{U}$ increase, starting at $\bar{U} = 0$. Initially, i.e. for small $\bar{U}$, the participation constraint remains slack. Then, at a threshold value $\bar{U}^*$, it becomes binding. If, from $\bar{U}^*$ on, $\bar{U}$ is further increased, the principal will have to increase at least one of the tournament-prizes in order to prevent the agents from quitting the job. It is then always profitable to choose a higher winner-prize and to leave the loser-prize unchanged. This leads to a higher prize-spread and less free-riding.

(iii) If $\bar{U}$ is so high that the participation constraint is binding, while the limited liability constraint becomes slack, the first-best solution will be achieved. In order to make the agents sign the contract, tournament prizes have to be chosen so high that limited liability does not constrain the optimal tournament contract. As a consequence, the principal's maximisation problem is the same as under unlimited liability, and, hence, an efficient solution is induced. Note that in this case the agents' reservation utilities are so high that the principal's profit is negative. He might thus be interested in cancelling the tournament to raise profit to zero.

5.3 Change in the principal's objective function

In this subsection, the principal is assumed to care only for the output of the best team, therefore he maximises $\operatorname{Max}\{y_1, y_2\} - w_l - l_l$. This change in the principal's objective function does not affect the second stage of the tournament. There is still a symmetric equilibrium with identical efforts of the agents. Hence, the two teams produce same output, and the principal's maximisation problem is given by

$$\begin{aligned} &\operatorname*{Max}_{w_l, l_l, e_l} \left[(n \cdot e_l)^\gamma - w_l - l_l \right] \\ &s.t.: \frac{(w_l - l_l) \cdot \gamma}{4 \cdot n^2 \cdot \delta} = (e_l)^\delta, \\ &\frac{w_l + l_l}{2 \cdot n} - (e_l)^\delta \geq 0, \\ &w_l, l_l \geq 0. \end{aligned} \tag{16}$$

Comparing this problem with the original problem (13), it becomes clear that the principal's gain from increasing the winner-prize is smaller in (16) than in (13), while the costs are identical. As a result, the winner-prize decreases compared to section 4 and the free-rider problem becomes even more intense.

This result is not very surprising. If a principal is only interested in the highest output, a tournament will clearly be inefficient, even under unlimited liability. Although the lowest output is totally worthless for the principal, in a tournament two teams compete and produce output. Thus, the principal pays for the production of output that he attaches no value to. It is then clear that he prefers to further decrease incentive strength so that the results from Proposition 2 are shifted even more in favour of the piece-rate scheme. Finally, notice that this inefficiency is not new, but familiar from the literature on patent race games (see e.g. Loury (1979)), where it has been termed 'duplication of effort'.

The results of section 5 are summarised in the subsequent proposition:

Proposition 3: Free-riding in collective tournaments as a consequence of the limited liability assumption —

• *will become less significant, if the agents have complete bargaining power;*

• *may decrease, if the agents have a positive reservation utility;*

• *will become even more intense, if the principal only cares for the output of the best team.*

6. Concluding remarks

In the literature on moral-hazard, a collective tournament is proposed as a means of mitigating the free-rider problem. Such a tournament is said to increase within-team cooperation and, hence, to motivate the teams' members. The aim of this paper was to analyse the impact of an agent's liability on the tournament outcome. It was found that free-riding is eliminated under unlimited liability. In the more realistic case of limited liability, however, a collective tournament performs very poorly. In this case, the principal is unable to afford intensified competition and so prefers to tolerate free-riding. Comparing the tournament to a simple piece-rate scheme, it was shown that, under limited liability, the tournament even intensifies the free-rider problem.

Further, it was found that free-riding might become less severe, if bargaining power is reversed or the agents have a positive reservation utility. On the other hand, the free-rider problem will be intensified, if the principal is solely interested in the best team's performance.

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Appendix

(I) Derivation of the optimal piece-rate scheme in the case of unlimited liability

In the case of unlimited liability, the Lagrangian to the principal's maximisation problem is

$$L = (1-\alpha_u \cdot n)\cdot\left(n \cdot e_{pr,u}\right)^{\gamma} - f_u \cdot n + \lambda_1 \cdot \left(\frac{\alpha_u \cdot \gamma \cdot (n)^{\gamma-1}}{\delta} - \left(e_{pr\,u}\right)^{\delta-\gamma}\right)$$

$$+\lambda_2 \cdot \left(f_u + \alpha_u \cdot \left(n \cdot e_{pr\,u}\right)^{\gamma} - \left(e_{pr\,u}\right)^{\delta}\right)$$

with the corresponding first-order conditions

$$\text{(a)}\quad \frac{\partial L}{\partial \alpha_u} = -n \cdot \left(n \cdot e_{pr,u}\right)^{\gamma} + \lambda_1 \cdot \frac{\gamma \cdot (n)^{\gamma-1}}{\delta} + \lambda_2 \cdot \left(n \cdot e_{pr\,u}\right)^{\gamma} \overset{!}{=} 0$$

(b) $$\frac{\partial L}{\partial e_{pr,u}} = (1-\alpha_u \cdot n)\cdot\gamma\cdot\left(n\cdot e_{pr,u}\right)^{\gamma-1}\cdot n-\lambda_1\cdot(\delta-\gamma)\cdot\left(e_{pr,u}\right)^{\delta-\gamma-1}$$
$$+\lambda_2\cdot\left(\alpha_u\cdot\gamma\cdot\left(n\cdot e_{pr,u}\right)^{\gamma-1}\cdot n-\delta\cdot\left(e_{pr,u}\right)^{\delta-1}\right)\overset{!}{=}0,$$

(c) $$\frac{\partial L}{\partial f_u} = -n+\lambda_2 \overset{!}{=} 0,$$

(d) $$\frac{\alpha_u\cdot\gamma\cdot(n)^{\gamma-1}}{\delta} = \left(e_{pr,u}\right)^{\delta-\gamma},$$

(e) $$\lambda_2\cdot\left(f_u+\alpha_l\cdot\left(n\cdot e_{pr,u}\right)^{\gamma}-\left(e_{pr,u}\right)^{\delta}\right)=0$$

Inserting $\lambda_2 = n$ (follows from (c)) into (a), yields $\lambda_1 = 0$. Then, from equations (b), (d) and (e),
$f_u = \left(\frac{\gamma}{\delta}\right)^{\frac{\gamma}{\delta-\gamma}}\cdot\left(\frac{\gamma}{\delta}\cdot n^{\frac{\delta\cdot(\gamma-1)}{\delta-\gamma}} - n^{\frac{\gamma\cdot(\delta-1)}{\delta-\gamma}}\right) < 0, \alpha_u = 1$ and $e_{pr,u} = e_{fb}$ can be derived easily.

(II) Derivation of the optimal piece-rate scheme in the case of limited liability

Under limited liability, the Lagrangian to the principal's maximisation problem is

$$L = (1-\alpha_l\cdot n)\cdot\left(n\cdot e_{pr,l}\right)^{\gamma} - n\cdot f_l + \lambda_1\cdot\left(\frac{\alpha_l\cdot\gamma\cdot(n)^{\gamma-1}}{\delta}-\left(e_{pr,l}\right)^{\delta-\gamma}\right)$$
$$+\lambda_2\cdot\left(f_l+\alpha_l\cdot\left(n\cdot e_{pr,l}\right)^{\gamma}-\left(e_{pr,l}\right)^{\delta}\right)+\lambda_3\cdot f_l.$$

We thus obtain the following first-order conditions:

(a) $$\frac{\partial L}{\partial\alpha_l} = -n\cdot\left(n\cdot e_{pr,l}\right)^{\gamma}+\lambda_1\cdot\frac{\gamma\cdot(n)^{\gamma-1}}{\delta}+\lambda_2\cdot\left(n\cdot e_{pr,l}\right)^{\gamma}\overset{!}{=}0$$

(b) $$\frac{\partial L}{\partial e_{pr,l}} = (1-\alpha_l\cdot n)\cdot\gamma\cdot\left(n\cdot e_{pr,l}\right)^{\gamma-1}\cdot n-\lambda_1\cdot(\delta-\gamma)\cdot\left(e_{pr,l}\right)^{\delta-\gamma-1}$$
$$+\lambda_2\cdot\left(\alpha_l\cdot\gamma\cdot\left(n\cdot e_{pr,l}\right)^{\gamma-1}\cdot n-\delta\cdot\left(e_{pr,l}\right)^{\delta-1}\right)\overset{!}{=}0,$$

(c) $\frac{\partial L}{\partial f_l} = -n + \lambda_2 + \lambda_3 \overset{!}{=} 0$

(d) $\frac{\alpha_l \cdot \gamma \cdot (n)^{\gamma-1}}{\delta} = (e_{pr,l})^{\delta-\gamma}$,

(e) $\lambda_2 \cdot \left(f_l + \alpha_l \cdot (n \cdot e_{pr,l})^{\gamma} - (e_{pr,l})^{\delta} \right) = 0,$

(f) $\lambda_3 \cdot f_l = 0.$

Equation (d) is equivalent to $\frac{\alpha_l \cdot \gamma \cdot (n)^{\gamma-1}}{\delta} \cdot (e_{pr,l})^{\gamma} - (e_{pr,l})^{\delta} = 0$. From this condition, it follows that the participation constraint never binds in equilibrium, as $\frac{\gamma \cdot (n)^{\gamma-1}}{\delta} < (n)^{\gamma}$. Hence, we have $\lambda_2 = 0$, which leads to $\lambda_3 = n$ and $f_1 = 0$. It is then easy to derive the remaining parameters $\alpha_l = \frac{\gamma}{\delta \cdot n}$ and $e_{pr,l} = \left(\frac{\gamma^2 \cdot n^{\gamma-2}}{\delta^2} \right)^{\frac{1}{\delta-\gamma}}$.

(III) Derivation of the tournament solution in the case of unlimited liability

Here, the Lagrangian to the principal's maximisation problem is given by

$$L = 2 \cdot (n \cdot e_u)^{\gamma} - w_u - l_u + \lambda_1 \cdot \left((w_u - l_u) \cdot \gamma - (e_u)^{\delta} \cdot 4 \cdot n^2 \cdot \delta \right) + \lambda_2 \cdot \left(\frac{w_u + l_u}{2 \cdot n} - (e_u)^{\delta} \right)$$

As first-order conditions we get:

(a) $\frac{\partial L}{\partial e_u} = 2 \cdot \gamma \cdot (n \cdot e_u)^{\gamma-1} \cdot n - \lambda_1 \cdot 4 \cdot (e_u)^{\delta-1} \cdot n^2 \cdot \delta^2 - \lambda_2 \cdot (e_u)^{\delta-1} \cdot \delta \overset{!}{=} 0$

(b) $\frac{\partial L}{\partial w_u} = -1 + \lambda_1 \cdot \gamma + \lambda_2 \cdot \frac{1}{2 \cdot n} \overset{!}{=} 0$

(c) $\frac{\partial L}{\partial l_u} = -1 - \lambda_1 \cdot \gamma + \lambda_2 \cdot \frac{1}{2 \cdot n} \overset{!}{=} 0$

(d) $(w_u - l_u) \cdot \gamma = (e_u)^{\delta} \cdot 4 \cdot n^2 \cdot \delta$

(e) $\lambda_2 \cdot \left(\frac{w_u + l_u}{2 \cdot n} - (e_u)^{\delta} \right) = 0$

Summation of (b) and (c) yields $\lambda_2 = 2 \cdot n$. It directly follows (from (b)) that $\lambda_1 = 0$. Knowing these multiplier values, simultaneous solution of (a), (d) and (e) yields the optimal tournament-prizes and efforts under unlimited liability.

(IV) Derivation of the tournament solution in the case of limited liability:

If the loser-prize is set equal to zero, the agents will receive a rent, i.e. an agent's expected utility exceeds his reservation utility. It follows that $l_1 = 0$ is optimal for the principal. A higher loser-prize increases the principal's wage costs and decreases the prize spread and, thus, incentive strength. Consequently, a strictly positive loser-prize cannot be optimal. With $l_1 = 0$, the Lagrangian to the principal's maximisation problem is

$$L = 2 \cdot (n \cdot e_1)^{\gamma} - w_1 + \lambda_1 \cdot \left(w_1 \cdot \gamma - (e_1)^{\delta} \cdot 4 \cdot n^2 \cdot \delta\right) + \lambda_2 \cdot \left(w_1 - 2 \cdot n \cdot (e_1)^{\delta}\right)$$

The corresponding first-order conditions are:

(a) $$\frac{\partial L}{\partial e_1} = 2 \cdot \gamma \cdot (n \cdot e_1)^{\gamma-1} \cdot n - \lambda_1 \cdot 4 \cdot (e_1)^{\delta-1} \cdot n^2 \cdot \delta^2 - \lambda_2 \cdot 2 \cdot (e_1)^{\delta-1} \cdot n \cdot \delta \overset{!}{=} 0$$

(b) $$\frac{\partial L}{\partial w_1} = -1 + \lambda_1 \cdot \gamma + \lambda_2 \overset{!}{=} 0$$

(c) $$w_1 \cdot \gamma = (e_1)^{\delta} \cdot 4 \cdot n^2 \cdot \delta$$

(d) $$\lambda_2 \cdot \left(w_1 - 2 \cdot n \cdot (e_1)^{\delta}\right) = 0$$

Transforming equation (c) yields $w_1 = (e_1)^{\delta} \cdot \dfrac{8 \cdot n^2}{\gamma}$. Since $\dfrac{4 \cdot n^2 \cdot \delta}{\gamma}$ exceeds $2 \cdot n$, it follows (from (d)) that $\lambda_2 = 0$.[16]

With $\lambda_2 = 0$, the solution $\lambda_1 = \dfrac{1}{\gamma}$, $e_1 = \left(\dfrac{n^{\gamma-2} \cdot \gamma^2}{2 \cdot \delta^2}\right)^{\frac{1}{\delta-\gamma}}$ and $w_1 = \dfrac{4 \cdot n^2 \cdot \delta}{\gamma} \cdot \left(\dfrac{n^{\gamma-2} \cdot \gamma^2}{2 \cdot \delta^2}\right)^{\frac{\delta}{\delta-\gamma}}$

can be derived easily.

(V) Derivation of the tournament solution in the case of reversed bargaining power:

In the case of reversed bargaining power, the principal determines the tournament prizes in order to maximise the agents' utilities. Thereby, he has to consider his zero-profit condition, the agents' incentive constraints and the limited liability constraint. The Lagrangian to the maximisation problem is thus given by

$$L = w_1 + l_1 - 2 \cdot n \cdot (e_1)^{\delta} + \lambda_1 \cdot \left(2 \cdot (n \cdot e_1)^{\gamma} - w_1 - l_1\right) + \lambda_2 \cdot \left((e_1)^{\delta} \cdot 4 \cdot n^2 \cdot \delta - (w_1 - l_1)\, \gamma\right) + \lambda_3 \cdot l_1 .$$

The first-order conditions are

(a) $$\frac{\partial L}{\partial e_1} = -2 \cdot n \cdot \delta \cdot (e_1)^{\delta-1} + \lambda_1 \cdot 2 \cdot \gamma \cdot n^{\gamma} \cdot (e_1)^{\gamma-1} + \lambda_2 \cdot 4 \cdot n^2 \cdot \delta^2 \cdot (e_1)^{\delta-1} \overset{!}{=} 0$$

(b) $\frac{\partial L}{\partial w_1} = 1 - \lambda_1 - \lambda_2 \cdot \gamma \overset{!}{=} 0$

(c) $\frac{\partial L}{\partial l_1} = 1 - \lambda_1 + \lambda_2 \cdot \gamma + \lambda_3 \overset{!}{=} 0$

(d) $(w_1 - l_1) \cdot \gamma = (e_1)^{\delta} \cdot 4 \cdot n^2 \cdot \delta$

(e) $2 \cdot (n \cdot e_1)^{\gamma} - w_1 - l_1 = 0$

(f) $\lambda_3 \cdot l_1 = 0$

We distinguish between two possible cases, $l_1 > 0$ and $l_1 = 0$. First, suppose that $l_1 > 0$: As $\lambda_3 = 0$, the optimal effort is the same as in the case of unlimited liability, i.e., the effort is $e_1 = \left(\frac{n^{\gamma-1} \cdot \gamma}{\delta} \right)^{\frac{1}{\delta-\gamma}} = e_{fb}$. However, the effort together with equations (d) and (e) implies that $l_1 < 0$, contradicting the initial assumption. Hence, $l_1 = 0$ must hold. With $l_1 = 0$, one can use equations (d) and (e) to show that the optimal effort in the case of reversed bargaining power is $e_1 = \left(\frac{n^{\gamma-2} \cdot \gamma}{2 \cdot \delta} \right)^{\frac{1}{\delta-\gamma}}$, which is higher than the effort in the original tournament under limited liability.

(VI) Derivation of the tournament solution in the case of a strictly positive reservation utility:

If the agents' reservation utilities become strictly positive (i.e. $\overline{U} > 0$), the Lagrangian to the principal's maximisation problem will change to

$$L = 2 \cdot (n \cdot e_1)^{\gamma} - w_1 - l_1 + \lambda_1 \cdot \left((w_1 - l_1) \cdot \gamma - (e_1)^{\delta} \cdot 4 \cdot n^2 \cdot \delta \right)$$
$$+ \lambda_2 \cdot \left(w_1 + l_1 - 2 \cdot n \cdot (e_1)^{\delta} - 2 \cdot n \cdot \overline{U} \right) + \lambda_3 \cdot l_1.$$

The first-order conditions are

(a) $\frac{\partial L}{\partial e_1} = 2 \cdot \gamma \cdot (n \cdot e_1)^{\gamma-1} \cdot n - \lambda_1 \cdot 4 \cdot (e_1)^{\delta-1} \cdot n^2 \cdot \delta^2 - \lambda_2 \cdot 2 \cdot (e_1)^{\delta-1} \cdot n \cdot \delta \overset{!}{=} 0$

(b) $\frac{\partial L}{\partial w_1} = -1 + \lambda_1 \cdot \gamma + \lambda_2 \overset{!}{=} 0$

(c) $\frac{\partial L}{\partial l_1} = -1 - \lambda_1 \cdot \gamma + \lambda_2 + \lambda_3 \overset{!}{=} 0$

(d) $(w_1 - l_1) \cdot \gamma = (e_1)^{\delta} \cdot 4 \cdot n^2 \cdot \delta$

$$\text{(e)}\quad \lambda_2 \cdot \left(w_1 + l_1 - 2 \cdot n \cdot (e_1)^{\delta} - 2 \cdot n \cdot \bar{U} \right) = 0$$

$$\text{(f)}\quad \lambda_3 \cdot l_l = 0$$

From (b) and (c), we obtain $2 \cdot \lambda_2 + \lambda_3 = 2$. Hence, it cannot be that both, the participation constraint and the limited liability constraint are slack. Three other cases remain:

1) If the limited liability constraint is binding, while the participation constraint is not, we will have $\lambda_2 = 0$ and $\lambda_3 = 2$. It is then easy to show that the solution is the same as in the initial tournament under limited liability.

2) If the participation constraint is binding and the limited liability constraint is not, it can be shown that $\lambda_3 = 0$ and $\lambda_2 = 1$. Then, it is easy to confirm that the solution is the same as in the initial tournament under unlimited liability.

3) If neither constraint is slack, $l_1 = 0$. From $w_1 - 2 \cdot n \cdot (e_1)^{\delta} = 2 \cdot n \cdot \bar{U}$ and $w_1 \cdot \gamma = (e_1)^{\delta} \cdot 4 \cdot n^2 \cdot \delta$, it follows that the effort is $e_1 = \left(\frac{\gamma \cdot \bar{U}}{2 \cdot n \cdot \delta} \right)^{\frac{1}{\delta}}$, which is increasing in $\bar{U}$.

Endnotes

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2. For a formal model highlighting the argument in the current paragraph see, e.g., the textbook by Laffont and Martimort (2002), p. 155-157.

3. The variable s is used to distinguish the case of unlimited liability (s=u) from the case of limited liability (s=l).

4. The restriction $\delta > 1$ is introduced to ensure the existence of all equilibria to be derived in this paper.

5. Output is measured in monetary terms. The restriction $\gamma \leq 1$ is introduced to ensure the existence of all equilibria to be derived in this paper.

6. Especially when considering research teams, one could also think that the principal is interested in maximising the function $\max\{y_1, y_2\} - w_s - l_s$. This is dealt with in section 5.

7. Note that here as well as in all following maximisation problems the second-order conditions are satisfied.

8. The derivations of this solution and the solutions to subsequent maximisation-problems are placed in the Appendix.

9. For a more detailed presentation of these problems, see Eswaran and Kotwal (1984).

10. Notice that the described problems do not arise under either the tournament or the piece-rate scheme.

11. Note that, under unlimited liability, the principal saves on wage costs by demanding an entrance fee from the agents.

12. See Nash (1950).

13. At first sight one could think that, although the agents have complete bargaining power, the principal might receive a (small, but) positive rent. The subsequent argument shows that this is not true. If, in the optimum, the principal received a positive rent, the agents could increase both tournament-prizes until the principal's rent becomes zero. Such an increase in prizes is always profitable for the agents and, since the limited liability constraint only gives a lower but no upper boundary for prizes, it is always feasible.

14. Note that there exists no equilibrium where both constraints are slack. If this were the case, the principal could marginally decrease the loser-prize. This would increase his profit without violating the constraints. Hence, such a deviation would always be profitable for the principal, contradicting the assumption that we initially considered an equilibrium.

15. Note that, under the different objective function, the maximisation problem under the piece-rate scheme is still given by (9).

16. From $\frac{4 \cdot n^2 \cdot \delta}{\gamma} > 2 \cdot n$, it also follows that the participation constraint is slack.

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